

S100 29 and 30

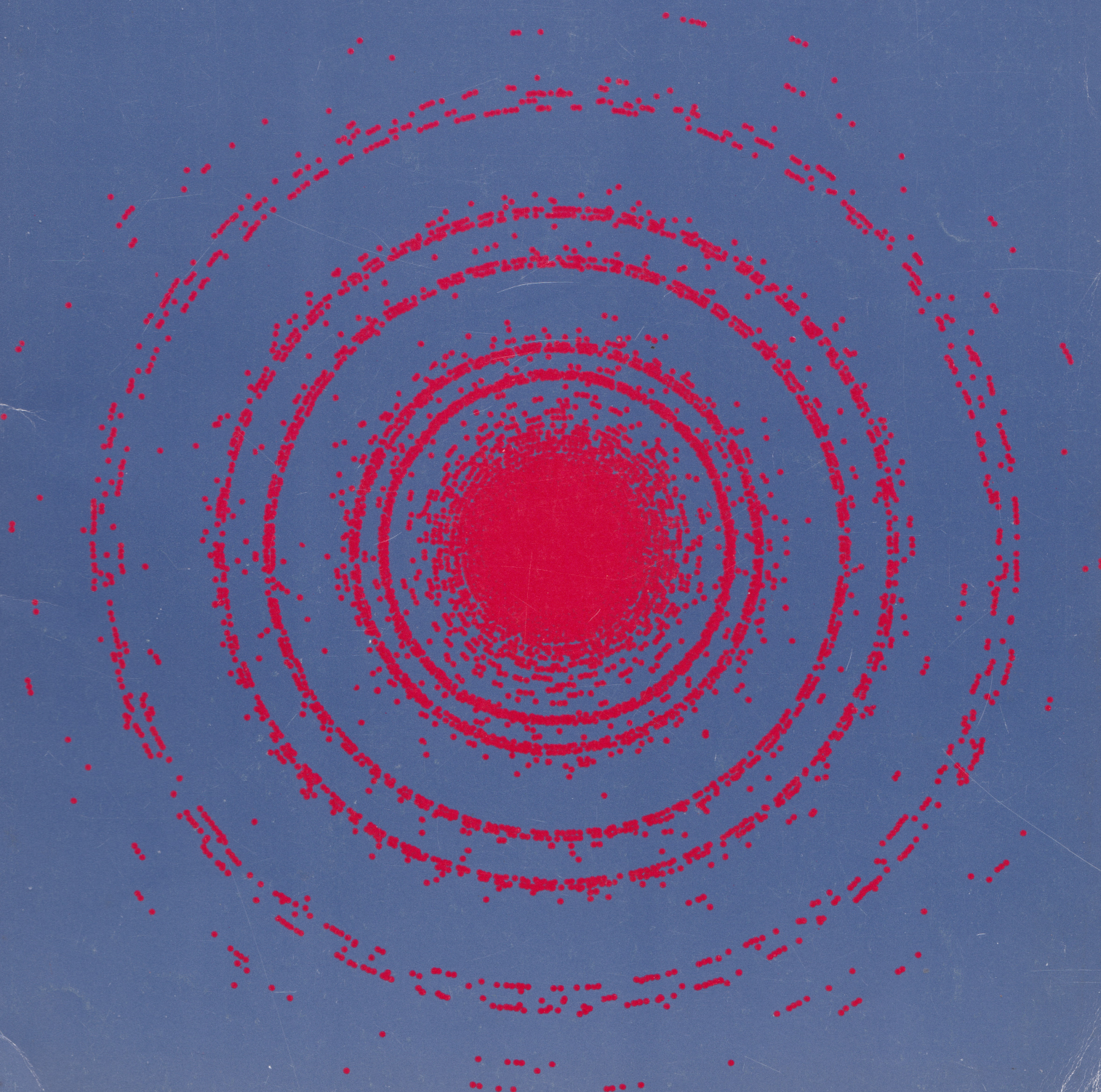
THE OPEN UNIVERSITY



Science Foundation Course Units 29 and 30

Quantum Theory

Quantum Physics and the Atom





The Open University

Science Foundation Course Unit 29

QUANTUM THEORY

Prepared by the Science Foundation Course Team

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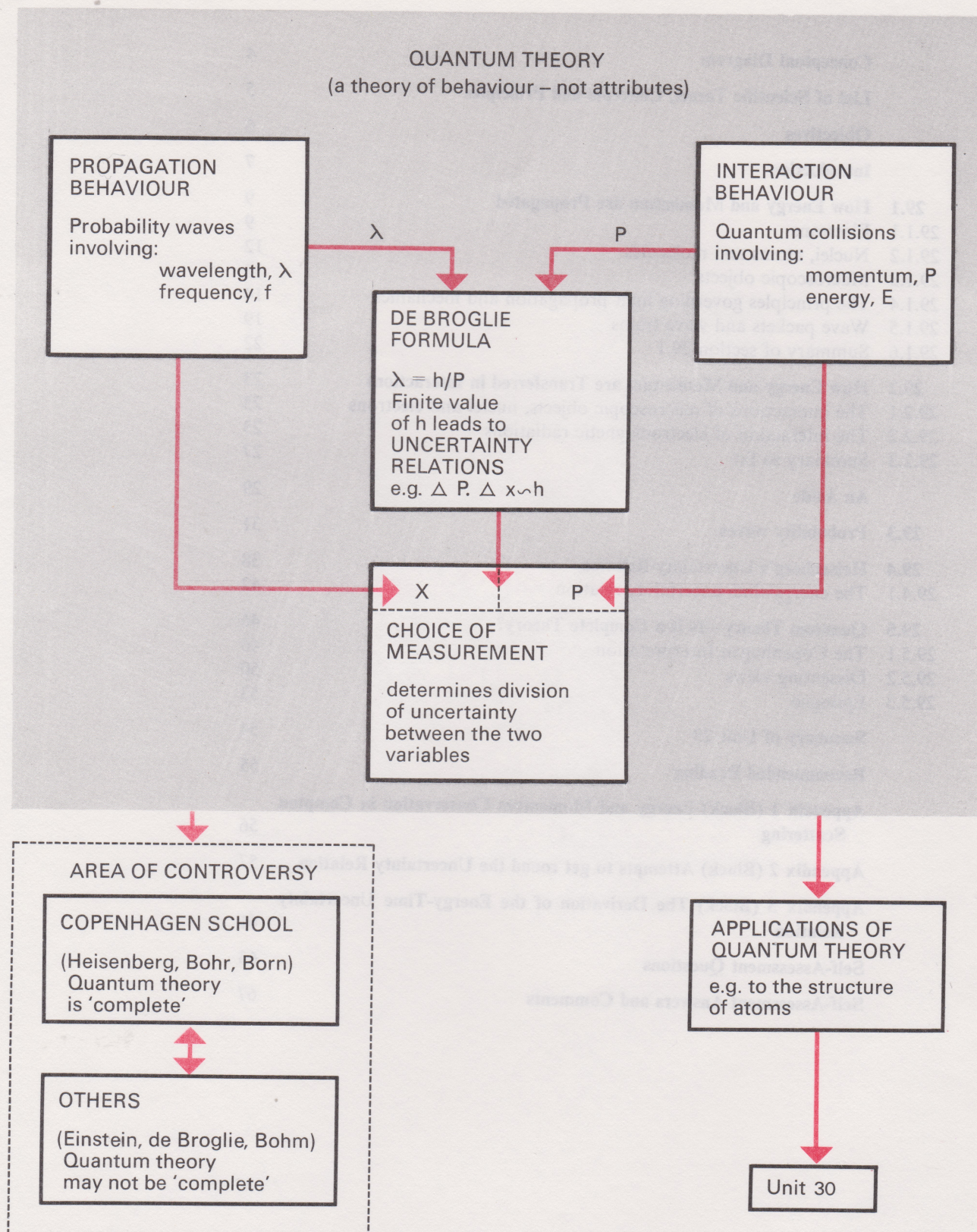


Table A

List of Scientific Terms, Concepts and Principles used in Unit 29

Taken as pre-requisites			Introduced in this Unit			
1	2	3	4			
Assumed from general knowledge	Introduced in a previous Unit	Unit No.	Developed in this Unit	Page No.	Developed in a later Unit	Unit No.
	electromagnetic radiation	2	de Broglie formula, $\lambda = h/p$	11		
	velocity	3	Planck's constant	11		
	electromagnetic, gravitational, and nuclear forces	4	Fermat's Principle of Least Time	16		
	molecules	5	action	18		
	crystals	5	all motion is wave motion	18		
	proton, electron, nucleus, atom	6	Maupertuis' Principle of Least Action	18		
	photon	6	wave packet	19		
	wavelength	2	photo-electric effect	23		
	frequency	2	Compton scattering of electro-magnetic radiation	26		
	momentum	3	all interactions governed by quantum behaviour	27	all interactions governed by quantum behaviour	32
	kinetic and potential energy	4	quantum	27	probability waves	30
	reflection	22	probability waves	31	uncertainty relations	31, 32
	refraction	22	uncertainty relations	38		
	diffraction	28	uncertainty principle	42		
	diffraction grating	28	Copenhagen Interpretation of quantum theory	46		
	interference	28	strict causality	46		
	Young's double-slit experiment	28				
	polarization	28				
	intensity of wave	28				
	size of diffraction dependent on wavelength	28				
	energy and momentum conservation	3, 4				
	laws of reflection and refraction	22				
	Huygens's Principle of secondary wavelets	22				
	Newton's laws of motion	3				

Objectives

When you have completed the work for this Unit, you should be able to:

- 1 Define or recognize adequate definitions of, or distinguish between true and false statements concerning each of the terms, concepts and principles in column 3 of Table A.
- 2 For the case of electron diffraction at a crystal surface, determine the wavelength of the beam given the separation of the atoms in the crystal surface and the angle of diffraction.
- 3 State de Broglie's formula. Given Planck's constant and the momentum (or wavelength) of a beam, calculate the wavelength (or momentum). Given a set of data from an experiment involving diffraction, check the validity of the de Broglie formula.
- 4 Distinguish between true and false statements concerning modes of propagation. Apply the principles of Fermat and Maupertuis to solve simple problems involving the determination of the paths of light and of particles.
- 5 Distinguish between true and false statements concerning the mode of interaction of quanta. Describe what is meant by Compton scattering and the photoelectric effect. In the case of the latter, state what effects are produced by changes in the intensity and frequency of the incident light, and in the nature of the metal surface. Check that a given set of experimental data is in accord with these statements.
- 6 Given a description of a specific experimental arrangement, identify the qualitative information it provides on the propagation and interaction aspects of the behaviour of radiation.
- 7 Explain how a wave packet of finite length can be built up from the superposition of continuous wave trains of differing wavelength. Relate this idea to Heisenberg's uncertainty relation: $\Delta p \Delta x \sim h$.
- 8 Explain what is meant by the uncertainty principle. Apply it to various experimental situations to explain why it is impossible to measure simultaneously and precisely both the position and momentum of an object, or to make a precise determination of the energy of a system if there is only a finite time available for the measurement.
- 9 Summarize the Copenhagen Interpretation of quantum theory and give some of the arguments that have been advanced both in its favour and against it. Apply it to various situations.
- 10 Write an essay tracing out the logical development of the idea of probability waves using basic experimental data on wave propagation and quantum interaction as a starting point.

Introduction

The ideas you will be introduced to this week will undoubtedly appear very strange. Physicists worked for the whole of the first quarter of this century unravelling the mystery of the quantum. Because of the conceptual difficulties involved in this subject it has long been customary to reserve it for the later stages of a degree course in physics. In recent years, however, many universities have experimented by bringing quantum theory forward in the curriculum and we shall follow their example.

Usually the subject is presented in a manner closely following its historical development. We shall be leaving aside this historical approach in favour of one that, to us at least, appears more straightforward and economical. The Unit as a whole should be regarded as a continuously developing line of argument. We shall ask certain questions of Nature, describe the answers given by experimental observation, and show how these observations logically lead to certain remarkable conclusions. As the argument unfolds, follow closely each step. In the process we shall challenge some ideas which at present appear to you to be plain common sense. Remember Einstein's description of common sense as 'that layer of prejudices laid down in the mind prior to the age of eighteen'. This week we think you are in for some surprises—the world never seems quite the same once one knows about its quantum behaviour.

The ideas you will be introduced to this week will undoubtedly appear very strange. Physicists worked for the whole of the last quarter of the century unravelling the mystery of the quantum. Because of the enormous difficulties involved in this subject it has long been customary to reserve it for the last stages of a degree course in physics. In recent years, however, many universities have experimented by teaching quantum theory forward in the curriculum and we shall follow their example.

Usually the subject is presented in a manner closely following its historical development. We shall be leaving aside the historical approach in favour of one that to us at least appears more straightforward and economical. The link as a whole should be regarded as a continuously developing line of argument. We shall ask certain questions of Nature, describe the answers given by experimental observation, and show how these observations logically lead to certain remarkable conclusions. As the argument unfolds, follow closely each step. In the process we shall challenge some ideas which at present appear to you to be plain common sense. Remember Einstein's description of common sense as 'that layer of prejudices laid down in the mind prior to the age of eighteen'. This week we think you are in for some surprises—the world never seems quite the same once one knows about its quantum behaviour.

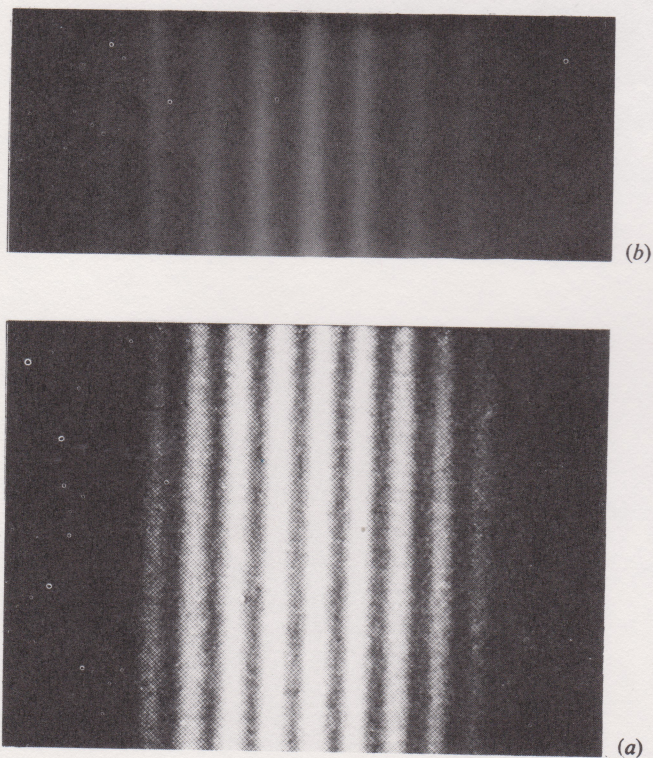


Figure 1

- (a) A double-slit interference pattern for electrons.
 (b) A double-slit pattern for light, shown for comparison.
 (The electron pattern was obtained by C. Jönsson at the University of Tübingen.)

Section 1

29.1 How Energy and Momentum are Propagated

You have already been introduced to some of the properties of light. In Unit 22 you saw how the laws of reflection and refraction arose in a natural way from the wave theory. Last week you studied diffraction, interference and polarization—phenomena associated with wave behaviour.

As you know, the laws of reflection and refraction indicate the directions in which light will travel after it has struck a boundary between two media. The state of polarization of the incident beam has a bearing on how much light is reflected and how much refracted at the surface, i.e. how much is turned back and how much goes forward. The theories of diffraction and interference are concerned with calculating how much light travels in a given direction after passing through gaps or round obstacles.

Can you suggest what type of question is common to these different experimental situations?

These various phenomena are concerned with a single general type of question, namely: how is light propagated? Although this question is sometimes answered by invoking interference, or perhaps reflection or refraction, there is only one general answer to the question, and that is: *light is propagated as a wave*. We shall now broaden this investigation by asking: *how are other forms of energy propagated?*

29.1.1 Electrons

We begin by studying the behaviour of a beam of electrons.* The fact that in earlier Units we referred to the electron as a 'particle' probably means you already have some mental picture of the way electrons move in space and time. If this is so—forget it. There is only one way of knowing how electrons are propagated and that is to perform an experiment to find out. Indeed our whole approach in this Unit will be to present you with the experimental observations first, and only then discuss the theoretical ideas that stem from them.

We begin by considering an experiment in which electrons from an electron gun are directed at two parallel slits in a barrier, much in the same way as light was shone onto parallel slits in Young's interference experiment. The electrons are subsequently recorded on a photographic plate placed on the opposite side of the barrier. The rather surprising outcome of this experiment (surprising if you *did* picture an electron as a tiny billiard ball) is shown in Figure 1. As you can see, there is essentially no difference between the intensity distribution of the electrons and that obtained with light in the conventional form of the experiment. In Unit 28 it was emphasized that interference phenomena such as this could only be described in terms of a wave theory. The alternating maximum and minimum intensities of the electrons are therefore a clear indication that

* Refer back to section 5.3 of Unit 2, if you have forgotten how a beam of free electrons can be produced.

in order to determine the distribution of scattered electrons, it is necessary to use the mathematical analysis associated with wave properties. It is difficult to perform Young's interference experiment using electrons because the wavelength of the beam is found to be very tiny. In order to produce an appreciable separation of the maxima and minima, the distance between the slits has to be very small too.

Do you remember the relation for the angular separation, θ , between two maxima in terms of the separation of the two slits, d , and the wavelength, λ ?

From Unit 28, the angular separation of two maxima is given by $\sin \theta = \lambda/d$. Thus if θ is to be appreciable and λ is small, d must also be small.

The experiments by Davisson and Germer and by G. P. Thomson, which first showed the wave behaviour of electrons, were not performed with man-made slits, but with a naturally occurring 'diffraction grating'. (In this Unit's TV programme you will be able to see a film of Dr. Germer demonstrating electron diffraction.) You will recall from Unit 5 that crystals consist of rows of regularly spaced atoms. The rows of atoms can be regarded as straight parallel lines from which light may be diffracted. A crystal surface therefore acts like a very finely ruled grating. When an electron beam is directed at such a surface it is preferentially scattered in certain directions giving diffraction maxima and minima.

You saw in Unit 28 that the wavelength of the light incident on a grating could be determined from the known spacing of the lines and the directions of the diffraction maxima. The wavelength of an electron beam can be estimated in the same way. In Figure 2, the difference, ΔD , between the

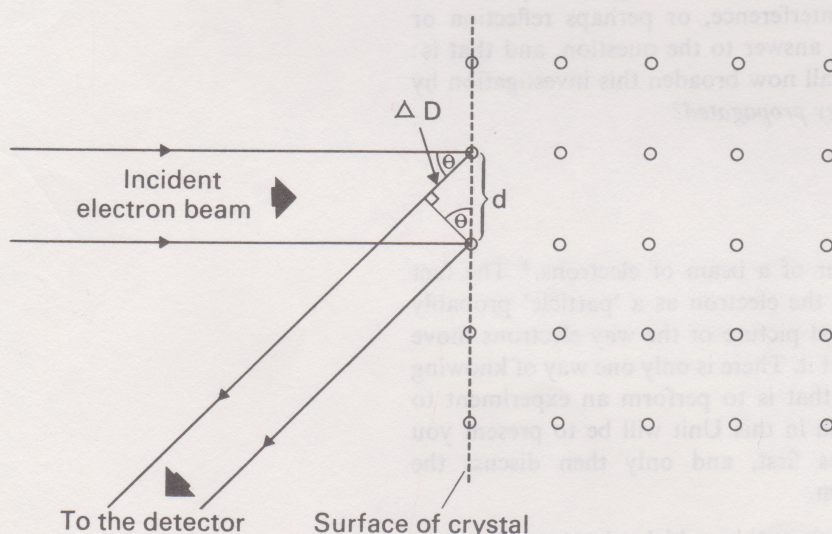


Figure 2 Electrons being scattered at the surface of a crystal.

lengths of the paths of the interfering rays must be a whole number of wavelengths if there is to be a maximum at angle θ . This angle is given in terms of ΔD and the separation of the atoms, d , by the equation

$$\sin \theta = \frac{\Delta D}{d}$$

For the n th maximum (n being zero for the maximum along the normal to the crystal surface), we have

$$\Delta D = n\lambda, \text{ and}$$

$$\sin \theta = \frac{n\lambda}{d}$$

Therefore

$$\lambda = \frac{d \sin \theta}{n}$$

For a crystal of a given substance, the separation of the atoms is known, so a measurement of θ allows λ to be determined.

(It should be borne in mind that the crystal represents not one diffraction grating but a whole set of plane gratings one behind the other. A complete description of the observed diffraction pattern is therefore more complicated than the one we have presented. Radiation penetrating to, and diffracting from, these additional layers will combine with that diffracted from the surface layer, and will sometimes lead to destructive interference and a diminishing of the expected maximum.)

What value does this type of experiment give for the wavelength associated with an electron beam? There is no unique answer; the angles of the diffraction maxima, and hence the wavelength, are found to vary as the momentum, p , of the electrons is changed. The momentum, depending as it does on the electron's velocity, may be controlled by the accelerating electric field in the electron 'gun', i.e. by the voltage on the accelerating anode. By observing how λ varies as a result of changes in the voltage, a very simple relationship is found:

$$\lambda = h/p \dots\dots\dots(2)$$

This is an exceedingly important formula: it is so important that we shall be requiring you to verify the relationship for yourself in one of the Home Experiments for this Unit. The relationship was originally proposed in 1924 by the French physicist, de Broglie, and so λ is called the *de Broglie wavelength* of the beam. The constant h is called *Planck's constant*, and you will come across it often in this Unit—indeed you were first introduced to it in Unit 6. Its value is: $h = 6.62 \times 10^{-34}$ joule seconds. (Note that it is not a dimensionless number—it has the units of energy $\times$ time.) This value is so very small that even though the momentum of an electron is also small, the wavelength of an electron beam is generally very short.

de Broglie wavelength
Planck's constant

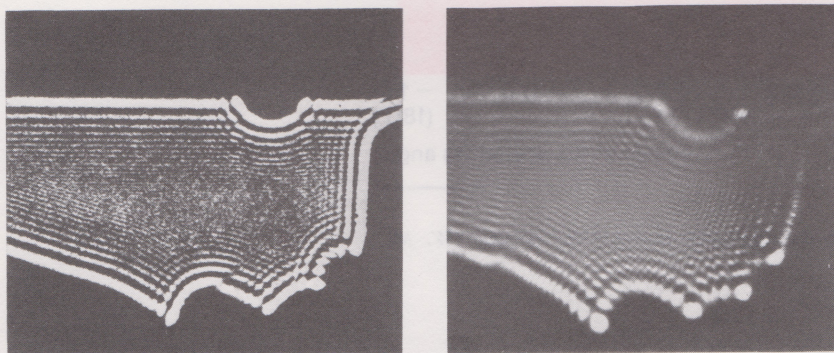


Figure 3 The picture on the right was formed by electrons emitted from a sharp point and passing through an aperture between tiny opaque crystals. For comparison, the other picture shows light being diffracted through an opening cut in a metal plate (Cavendish Laboratory).

We offer in Figure 3 just one final piece of evidence to convince you of the wave-like characteristics of electron propagation. Here electrons are diffracted at the edges of opaque crystals and, for comparison, light is shown diffracted through a similarly shaped opening; the similarities are striking.

Although earlier we spoke of electrons as particles, the conclusion to be drawn from the experimental evidence here presented is inescapable—in order to describe how electrons are *propagated* it is necessary to use wave theory.

29.1.2 Nuclei, atoms and molecules

We now move on to other types of radiation. How are beams of nuclei, e.g. protons or deuterons, propagated? Because nuclei are so much heavier than electrons, it is more difficult to produce a beam of them with very low momentum. If the de Broglie formula were to apply to them as well as to electrons, and if the same constant, h , were to be used in the formula, then clearly the wavelength of the beam of nuclei would in general be less than that of an electron beam; this would make it even more difficult to observe diffraction effects. However, diffraction *has* been observed for such beams, as can be seen in Figure 4. Here nitrogen nuclei act as

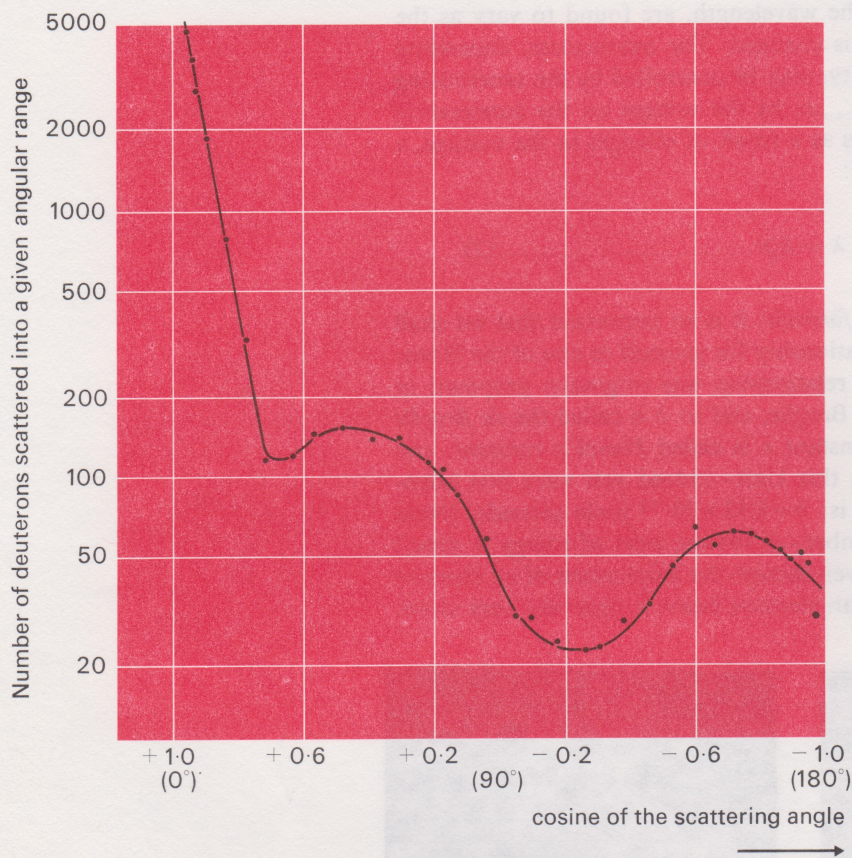


Figure 4 Angular distribution of deuterons elastically scattered by nitrogen (W. M. Gibson and E. E. Thomas).

obstacles in the way of a beam of deuterons (i.e. nuclei consisting of one neutron and one proton) and these cause the beam to be diffracted. The large peak in Figure 4 shows that most of the deuterons are only slightly affected and are still to be found approximately in the forward direction. There are, however, several lesser peaks indicating other likely directions—these are diffraction maxima. If you compare the curve of Figure 4 with that of Figure 5, showing the diffraction of light, you see the two have remarkably similar features.

The same diffraction phenomena have now also been observed for atoms and even molecules. These latter observations are significant because atoms and molecules are hardly to be considered as elementary quantities—they are groups of particles (electrons and nuclei) bound together. Even so, the group is found to be diffracted *as a complete unit*. Once again it is found that the diffraction pattern sizes depend upon momentum; de Broglie’s formula applies to all these various types of radiation.

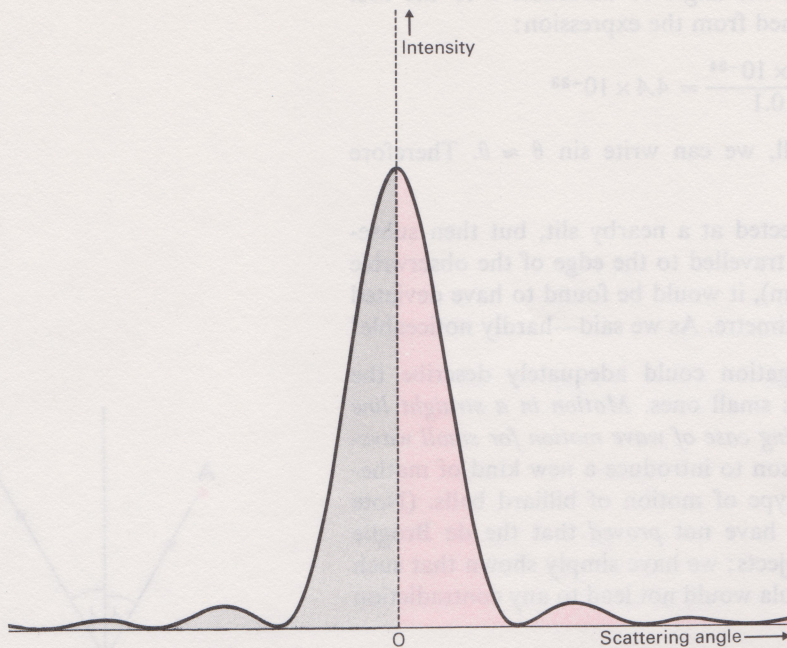


Figure 5 Intensity distribution of light diffracted through a slit, or round a small obstacle. The two halves of the symmetrical curve are shown in different colours for easier comparison with Figure 4 (where left and right scatterings were superimposed).

29.1.3 Macroscopic objects

But what if we extend the investigation still further? Suppose energy is being 'radiated' in the form of a stream of macroscopic (i.e. large) objects—for example, a stream of billiard balls? Surely that is taking things too far!

In the simplest case, where no forces act on the billiard balls, they continue moving in a straight line with constant speed. If they pass through a gap in a barrier, they continue moving in a straight line.

Is this necessarily different from the wave motion we have been discussing so far in this Unit? Do waves ever continue moving straight on having passed through a barrier?

In Figure 6, we show water waves passing through gaps of different sizes in a barrier; you first saw these pictures in Unit 2. When the wavelength is smaller than the dimensions of the gap, the wave is seen to continue moving in the forward direction and there is little evidence of any change of direction. Because the de Broglie wavelength of billiard balls would be very small compared with the size of any gap through which the balls could pass, a deviation due to diffraction would not be noticeable.

To stress this point, we take the case of a ball weighing 0.5 kg moving with speed 3 m s⁻¹. Its momentum, p , is given by $mv = 0.5 \times 3 = 1.5$ kg m s⁻¹. Applying de Broglie's formula we have:

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{1.5} = 4.4 \times 10^{-34} \text{ m}$$

Suppose the ball approaches a gap large enough for it to pass through.

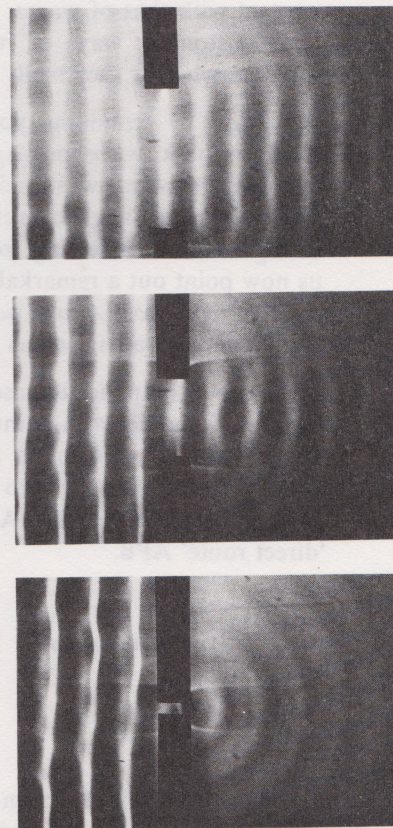


Figure 6 Photographs of water waves passing through a gap in a barrier. Diffraction becomes more apparent the smaller the size of the gap compared with the wavelength.

If the width of the gap is 0.1 m, the angle of deviation θ to the first diffraction minimum can be obtained from the expression:

$$\sin \theta = \frac{\lambda}{d} = \frac{4.4 \times 10^{-34}}{0.1} = 4.4 \times 10^{-33}$$

Because θ is obviously very small, we can write $\sin \theta \approx \theta$. Therefore $\theta = 4.4 \times 10^{-33}$ radians.

This means that if the ball is directed at a nearby slit, but then subsequently observed only after it has travelled to the edge of the observable universe (a distance of about 10^{25} m), it would be found to have deviated sideways a few millionths of a centimetre. As we said—hardly noticeable!

Thus the wave theory of propagation could adequately describe the motion of large objects as well as small ones. *Motion in a straight line can be regarded simply as the limiting case of wave motion for small wavelengths.* There is absolutely no reason to introduce a new kind of mathematics to be able to handle this type of motion of billiard balls. (Note that up to this point at least we have not *proved* that the de Broglie formula applies to macroscopic objects; we have simply shown that such an extension of the use of the formula would not lead to any contradiction with known behaviour.)

29.1.4 The principles governing light propagation and mechanics

But of course macroscopic objects do not always travel in straight lines—under the influence of gravity they move in curved trajectories. When faced with this kind of behaviour, it is customary to rely on Newton's laws of motion and introduce concepts like mass, force, gravitational attraction, etc.—a language that appears to have little or no point of contact with that of waves. In this section you will take a fresh look at your customary ways of regarding wave and particle propagation to discover whether they really are as different as they seem.

We begin by reminding you of what you have already learnt in this Course about wave propagation. In Unit 22, Huygens's method of secondary wavelets was used to prove the law that the angle of reflection was equal to the angle of incidence. Thus a ray of light going from A to B in Figure 7 and reflecting from the surface SS' takes the path AOB. Let us now point out a remarkable characteristic of this path; *it is impossible for light to go from A to B via a reflection at SS' any quicker than by AOB.* Neighbouring paths such as those illustrated in Figure 8 all take longer.

The same also applies to cases of refraction. Earlier you were shown that the path taken between points A and B in Figure 9 is such that angle r is related to angle i through Snell's law (Unit 22). But another way of looking at the problem is to say that the path AOB is the quickest route for light to travel between A and B. It is even quicker than by going the 'direct route' APB.

How can this be so?

The wave speed is lower in the second medium, so although the total distance travelled is less, the extra time spent moving slowly outweighs the advantage of the shorter distance; if the light remains longer in the medium in which its speed is greater it reaches B quicker.

Exactly the same considerations apply to waves moving in media with gradually changing properties. Light coming from the Sun, for example,

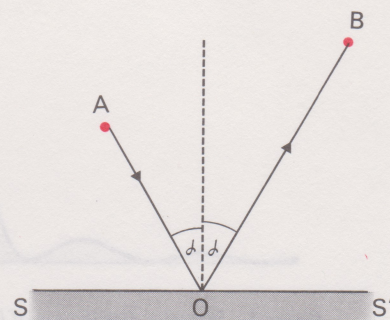


Figure 7 The path of a light ray passing from A to B via a reflection at the surface SS' .

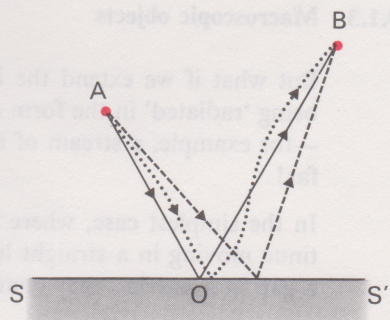


Figure 8 A selection of possible hypothetical routes from A to B via a reflection at the surface SS' .

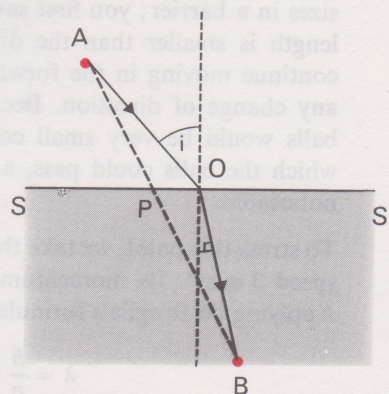


Figure 9 The path of a light ray passing from A to B via refraction at the surface SS' .

follows a curved path as it traverses the Earth's atmosphere (Fig. 10). The lower layers are generally denser, so the wave's speed decreases the deeper the penetration. It is quicker for the light to reach its destination on the Earth's surface by following the curved trajectory than for it to take the straight path and find itself spending longer travelling with lower speed in the denser atmosphere. The curved path of light is dramatically illustrated in Figure 11 which shows the Canigou Mountain (2 780 m) as seen 250 km away at Marseille. From this height and distance and the known radius of the Earth, it can be calculated that the peaks should be below the horizon.

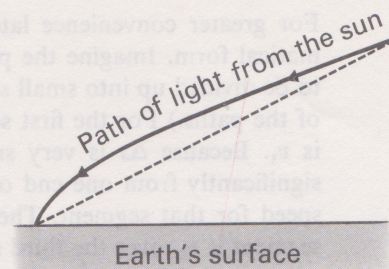


Figure 10 The path of a light ray passing from the Sun to the surface of the Earth. It is curved because the refractive index of the air changes with depth.

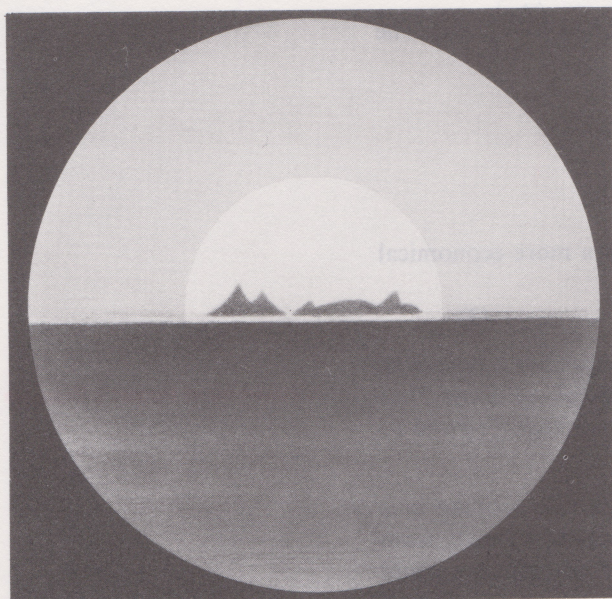
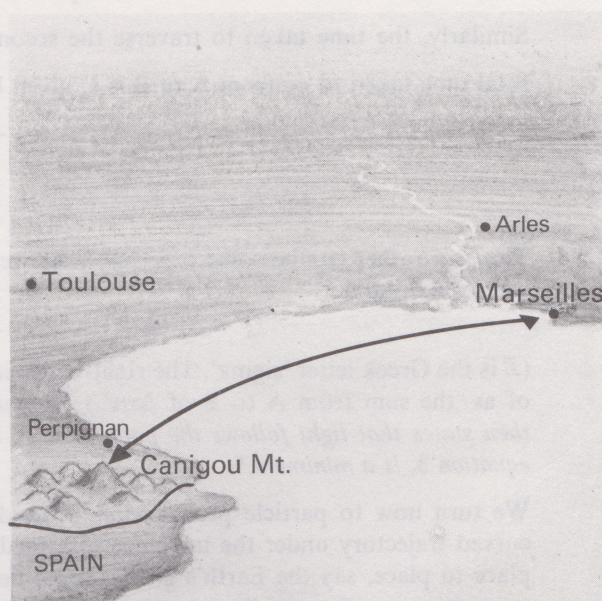


Figure 11 The Canigou Mountain, seen from Marseilles, silhouetted against the setting sun. The mountain should lie below the horizon and can only be seen because the light rays follow a curved path round the surface of the Earth. (The photograph was taken on the 12th February 1905.)



Thus, light moves between two points by the route that is quicker than any of the other routes nearby; this is *Fermat's Principle of Least Time*.^{*} With this powerful principle we are able to derive the laws of reflection and refraction and compute the curved trajectories of light rays moving in media of varying properties.

Fermat's Principle of Least Time

It is not too difficult to see how Fermat's Principle and Huygens's approach are related. You remember in Huygens's method, each point on a wave front is to be regarded as a source of secondary wavelets. As these wavelets travel, they interfere with each other. Where they interfere constructively the new wave front is formed; elsewhere they interfere destructively. The particular series of wavelets that allows the light to reach its destination by the quickest route obviously cannot be cancelled out by wavelets travelling via some other route, as the latter will take longer to arrive and will be too late to give destructive interference. Therefore the wavelets that keep ahead of 'the rest of the field' are certain to survive. In other words, the path taken up by the wave is the quickest route between any two points—which is Fermat's Principle.

^{*} P. Fermat (1601–1665), French mathematician.

For greater convenience later, let us express this principle in a mathematical form. Imagine the possible paths between A and B in Figure 12 to be divided up into small segments of length, Δs . (We show this for one of the paths.) For the first segment, the speed of light while traversing it is v_1 . Because Δs is very small we may assume that v_1 does not vary significantly from one end of the segment to the other: v_1 is the average speed for that segment. The average speed of the light over the second segment is v_2 , over the third segment v_3 , and so on. These various average speeds will differ from each other if the refractive index of the medium is changing.

The time, Δt_1 , taken to traverse the first segment, is given by

$$\Delta t_1 = \frac{\Delta s}{v_1}$$

Similarly, the time taken to traverse the second is $\Delta t_2 = \left(\frac{\Delta s}{v_2}\right)$, etc. The total time taken to go from A to B is t_A^B given by

$$t_A^B = \Delta t_1 + \Delta t_2 + \Delta t_3 + \dots$$

$$\text{i.e. } t_A^B = \frac{\Delta s}{v_1} + \frac{\Delta s}{v_2} + \frac{\Delta s}{v_3} + \dots$$

This is a rather cumbersome expression, so we adopt a more economical way of writing it

$$t_A^B = \Sigma_A^B (\Delta s/v) \dots \dots \dots (3)$$

(Σ is the Greek letter 'sigma'. The right-hand side of equation 3 is spoken of as 'the sum from A to B of $\Delta s/v$ '.) *Fermat's Principle of Least Time then states that light follows the path from A to B such that t_A^B , given by equation 3, is a minimum.**

We turn now to particle propagation. Consider a particle moving in a curved trajectory under the influence of a field that changes slowly from place to place, say the Earth's gravitational field; for example, we could take the case of a small stone thrown from one point on the Earth's surface to another. The trajectory of the stone could, of course, be worked out using Newton's laws of motion, but what is being sought here is a fresh approach to the problem—one that is more closely akin to the application of Fermat's Principle to light paths.

From energy conservation, it is known that as the stone rises and its potential energy increases, its kinetic energy decreases an equivalent amount (neglecting for simplicity the effects of air resistance). Bearing this in mind we ask whether the path of the stone is governed by Fermat's Principle of Least Time.

Do you expect the stone to take the quickest route between the two points?

The answer is no. If all one has to worry about is conserving energy, the quickest route would be the shortest—a horizontal straight line. In actual fact, of course, it is found that the stone has to be thrown upwards at an angle as in Figure 13. In this way, not only is the path longer than the straight line joining the two points, A and B, but the stone loses kinetic energy with height and therefore travels more slowly. Clearly a straight-forward application of Fermat's Principle does not work for particles!

However, a principle of sorts *does* exist for the paths of particles. Although not taking the path of shortest time, the stone nevertheless follows a

* Suggestion for further reading: for an extended account of Fermat's Principle and how it can be applied to various situations you might like to consult Chapter 26 of Feynman, Lectures on Physics, Vol. I, Addison Wesley (1963).

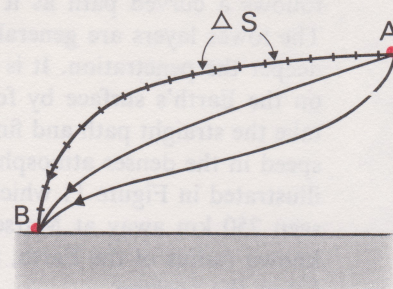


Figure 12 A selection of possible hypothetical routes from A to B through the air. One of them is shown divided into equal segments of length Δs .

a statement of Fermat's Principle

path with a unique characteristic. This is embodied in *Maupertuis's Principle of Least Action*.*

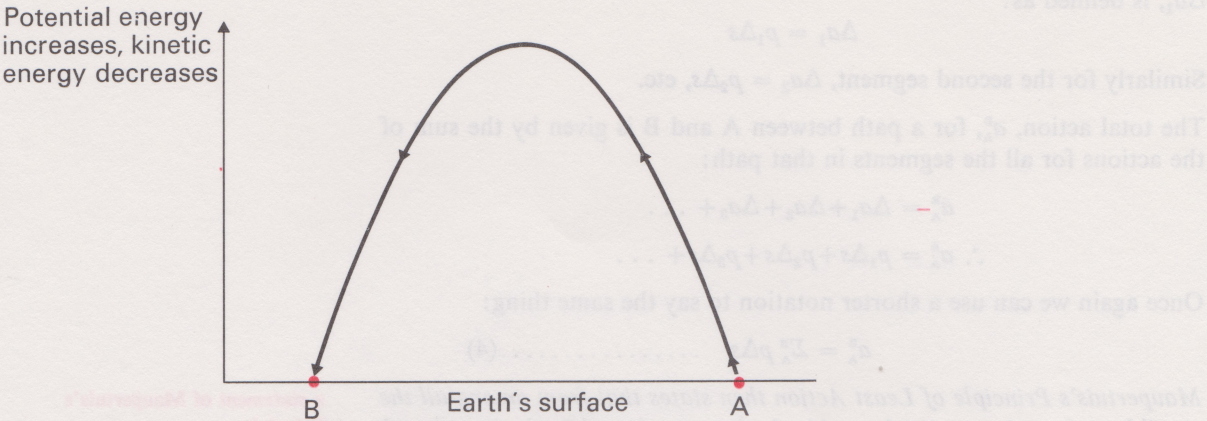


Figure 13 The path taken by a stone thrown from A to B (neglecting the effects of air resistance).

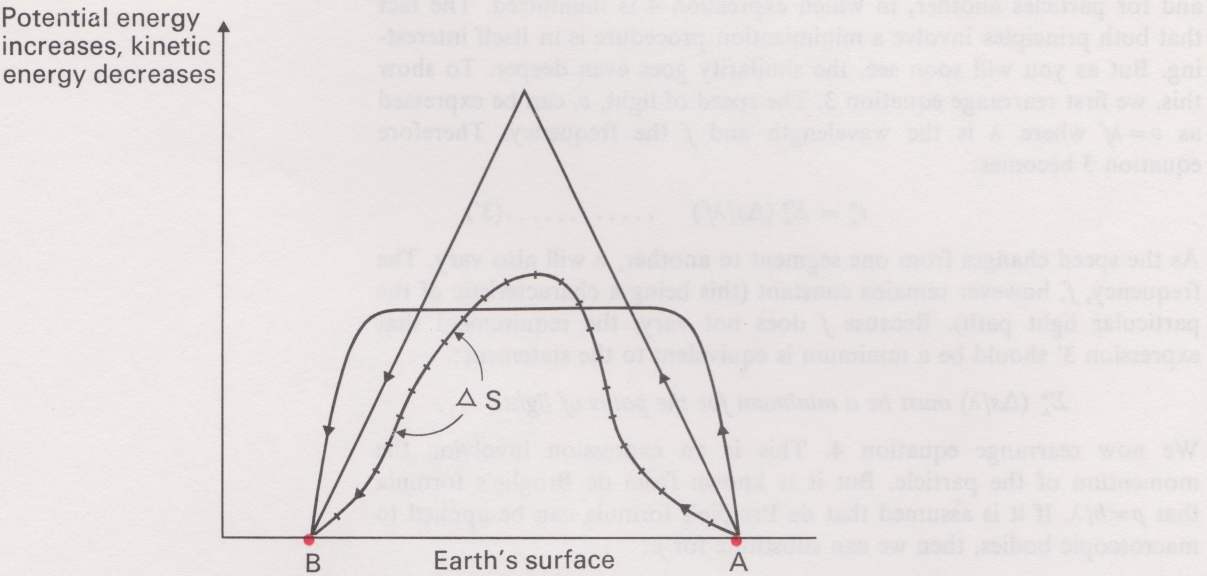


Figure 14 Possible hypothetical paths for a stone passing from A to B. One of them is shown divided into equal segments of length Δs .

From the knowledge that a stone has a certain total energy, all manner of possible paths may be drawn between A and B; we show a selection in Figure 14. From all of these trajectories Maupertuis's Principle singles out the right one. To apply the principle, each path is divided up into segments of length Δs , exactly as was done for the paths of light. The average momentum for the first segment is p_1 .**

* Maupertuis (1699–1759).
 ** This is obtained by noting that the total energy, E , is fixed and equals the sum of the kinetic ($\frac{1}{2}mv^2$) and potential (U) energy:

$$E = \frac{1}{2}mv^2 + U$$

The momentum, p , equals mv and can be expressed in terms of U and E :

$$p = mv = \sqrt{2m(E - U)}$$

The potential energy, U , is a function of the height of the particle, and hence a function of position. This means that p will also be a function of position.

Likewise the momentum for the second segment is p_2 , and so on.

We now define a quantity called *action*. For the first segment the action, Δa_1 , is defined as:

$$\Delta a_1 = p_1 \Delta s$$

Similarly for the second segment, $\Delta a_2 = p_2 \Delta s$, etc.

The total action, a_A^B , for a path between A and B is given by the sum of the actions for all the segments in that path:

$$\begin{aligned} a_A^B &= \Delta a_1 + \Delta a_2 + \Delta a_3 + \dots \\ \therefore a_A^B &= p_1 \Delta s + p_2 \Delta s + p_3 \Delta s + \dots \end{aligned}$$

Once again we can use a shorter notation to say the same thing:

$$a_A^B = \sum_A^B p \Delta s \quad \dots \dots \dots (4)$$

*Maupertuis's Principle of Least Action then states that from among all the possible trajectories a particle could take between A and B, the actual path is that for which a_A^B is a minimum.**

a statement of Maupertuis's Principle

So for light there is a principle involving the minimization of expression 3, and for particles another, in which expression 4 is minimized. The fact that both principles involve a minimization procedure is in itself interesting. But as you will soon see, the similarity goes even deeper. To show this, we first rearrange equation 3. The speed of light, v , can be expressed as $v = \lambda f$ where λ is the wavelength and f the frequency. Therefore equation 3 becomes:

$$t_A^B = \sum_A^B (\Delta s / \lambda f) \quad \dots \dots \dots (3')$$

As the speed changes from one segment to another, λ will also vary. The frequency, f , however remains constant (this being a characteristic of the particular light path). Because f does not vary, the requirement that expression 3' should be a minimum is equivalent to the statement:

$$\sum_A^B (\Delta s / \lambda) \text{ must be a minimum for the paths of light.}$$

We now rearrange equation 4. This is an expression involving the momentum of the particle. But it is known from de Broglie's formula that $p = h / \lambda$. If it is assumed that de Broglie's formula can be applied to macroscopic bodies, then we can substitute for p :

$$a_A^B = \sum_A^B (h / \lambda) \Delta s \quad \dots \dots \dots (4')$$

But h is a constant, so the requirement that expression 4' should be a minimum is the same as saying:

$$\sum_A^B (\Delta s / \lambda) \text{ must be a minimum for the paths of particles.}$$

Thus we come to the remarkable conclusion that THE TYPE OF MATHEMATICS USED TO DESCRIBE HOW PARTICLES ARE PROPAGATED IS THE SAME AS THAT USED TO DESCRIBE HOW LIGHT IS PROPAGATED.

all types of energy propagation governed by the same mathematics

* To be strictly accurate it is not necessary for either the time for a light path or the action for a path of a particle to be a minimum. It has to have a 'stationary' value. A minimum is one particular form of a stationary value; a maximum is another. We might also mention that in connection with the paths of particles one normally talks of Hamilton's Principle. This is more general in character than Maupertuis' Principle; it concerns not only those paths we have been considering along which the total energy is conserved, but also paths for which energy conservation would have to be violated in order for the particle to complete the course in the given time interval. None of this, however, need concern you; the more restricted Maupertuis' Principle is adequate for the argument being developed.

What is the physical interpretation of this powerful unifying principle? The quantity $(\Delta s/\lambda)$ is the length of a segment of the path expressed as a fraction of the wavelength of the radiation when traversing that segment. Therefore $\Sigma_A^B (\Delta s/\lambda)$ is the total length of the path expressed in wavelengths. *Thus any type of radiation chooses that path between two points that is characterized by the least number of wavelengths.*

any type of radiation chooses that path between two points that is characterized by the least number of wavelengths

This, then, is the fundamental principle that applies equally to light and to particles. Its power is really quite astonishing; e.g. it allows us to compute not only the curved path of the Sun's rays as they pass through the various layers of the Earth's atmosphere, but also the curved path of the Earth itself as it orbits the Sun.

(One small point that may still be worrying you: if the principles of Fermat and Maupertuis are essentially the same, why does the stone not travel between A and B by the same path as that followed by light? The reason is that Fermat's Principle takes the general principle concerning the least number of wavelengths (which applies to all kinds of radiation) and applies it to the special case of radiation having no rest-mass (Unit 4). For this particular type of radiation, and only this type, the path with the least number of wavelengths happens also to be the one for which the radiation takes the least time. Light is an example of such radiation.)

29.1.5 Wave packets and wave trains

If this idea of describing the propagation of objects in terms of the mathematics of waves is new to you, as it probably is, you may well find it takes a time to assimilate. For example, you may want to argue that macroscopic objects do not *look* like waves. Waves spread out through space, but particles are localized—their centres of gravity, for instance, have well-defined positions.

If this is how you are thinking, remember that there are different kinds of waves. There are continuous *wave trains*, consisting of long sequences of regularly spaced crests and troughs, and there are *wave packets*, consisting of a single irregular hump, like that which passes along a horizontal rope when the end is rapidly jerked upwards (Unit 22, Fig. 7).

wave trains
wave packets

Which type of wave do you think we ought to associate with a macroscopic object?

As we have said, a macroscopic object normally has a position that can be rather well specified. If one wants to use a mathematical wave to specify its position, it is natural to choose a wave that is also localized in space, i.e. a wave packet rather than a long wave train.

This being so, can you see a difficulty arising in connection with defining the de Broglie wavelength of the object?

This raises a problem. For an infinitely long wave train, the idea of a wavelength takes on a definite meaning—it is the characteristic distance between successive crests or troughs. But for an irregularly shaped packet, consisting perhaps of a single hump, how can one possibly speak of it as having a wavelength?

In order to answer this, we must introduce you to an interesting fact—wave packets of almost any shape can be regarded as the superposition of a selection of infinitely long wave trains. The basic idea is to choose

certain infinitely long wave trains, each with constant characteristic wavelength and amplitude, and allow them to interfere with each other. If the right choices of wavelength and amplitude have been made, then constructive interference takes place in a localized region (this gives the wave packet) and complete destructive interference everywhere else. There is a special mathematical procedure (called Fourier analysis) which enables the right selection of component waves to be made. We cannot go into it here, but at least we can give you a feel for how these waves interfere with each other.

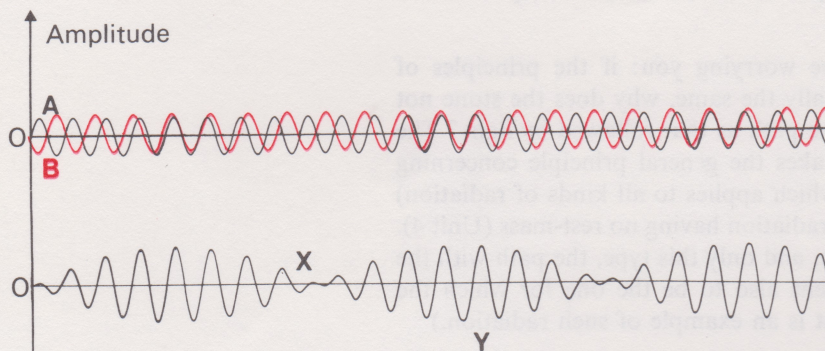


Figure 15 The two wave trains, A and B, interfere to produce the resultant disturbance shown above.

Figure 15 is a diagram you first came across in Unit 22 when the superposition principle was being discussed. Two wave trains of slightly different wavelength are shown repeatedly getting in and out of step. The resultant disturbance consists of a series of short-wavelength fluctuations and superimposed upon them a longer-wavelength variation giving rise to minima and maxima at, for example, X and Y respectively. At the minima the two waves are out of step, and at the maxima in step.

Thus the addition of only two waves has the marked effect of breaking up the regular wave trains into a series of wave packets. The length of the wave packets obviously depends on the distance between two points where the wave trains cancel each other, and this in turn depends upon their wavelengths. By choosing a different pair of waves, we could have produced wave packets of different lengths. The bigger the difference in wavelength, the shorter the distance over which they get out of step and hence the shorter the length of the wave packets.

By adding more wave trains, further modifications can be made. It is possible to add waves that enhance one particular wave packet and get out of step at progressively longer distances. In this way, all the wave packets can be wiped out except one. In Figure 16 you can see that as the spread in wavelengths increases, so the wave packet becomes sharper and more pronounced.

It is clear that *a wave packet does not have a precisely defined wavelength; it is a mixture of waves of different wavelengths*. Only if certain waves with approximately the same wavelength dominate all other waves, i.e. if these have much larger amplitudes than the others, is it possible to speak of even an approximate wavelength for the packet. *This property of wave packets assumes a crucial significance later in the Unit.*

a wave packet does not have a precisely defined wavelength

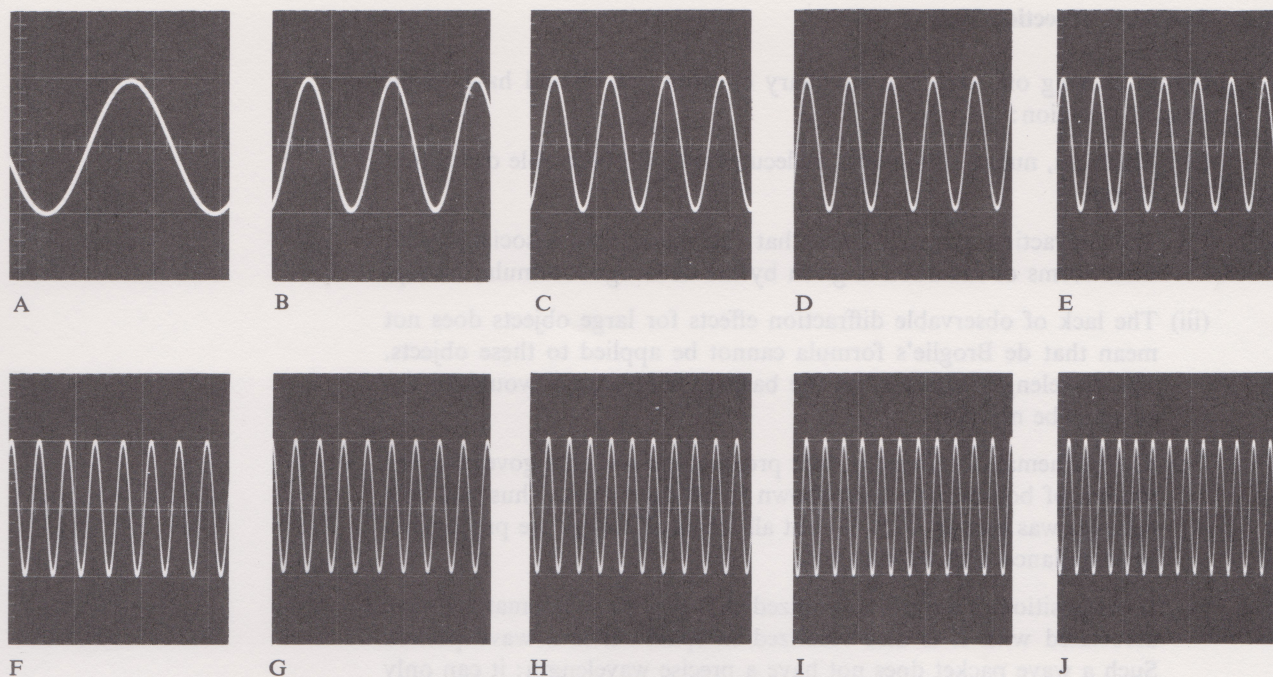
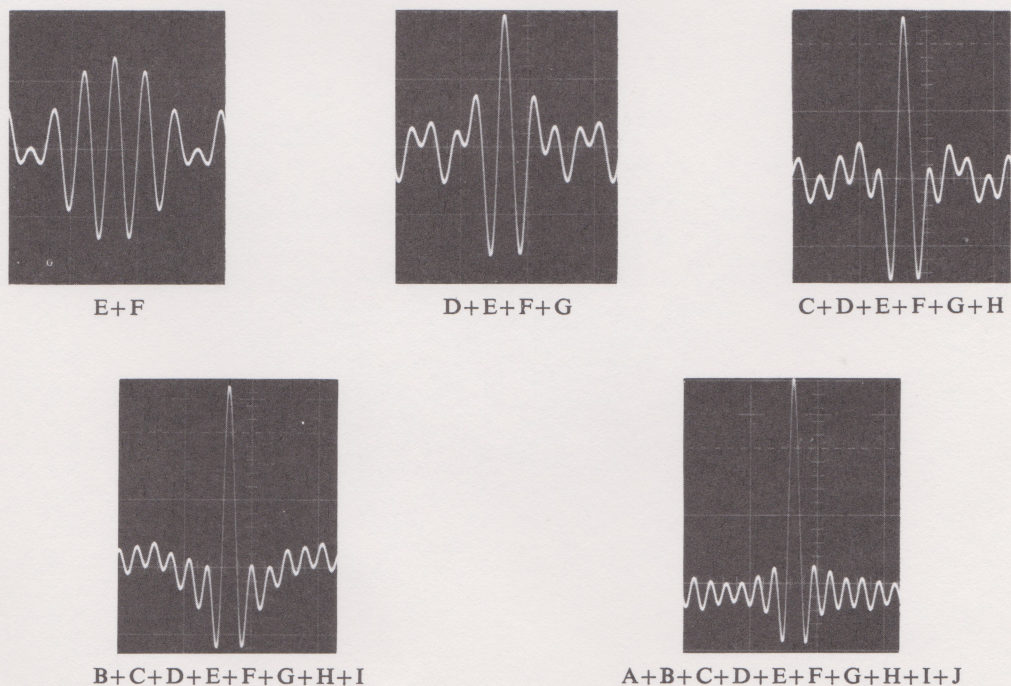


Figure 16

(a) Ten wave trains of different wavelength displayed on the screen of an oscilloscope.



(b) The waves are added progressively. As the spread in wavelength of the constituent waves increases, so the wave packet becomes narrower.

29.1.6 Summary of section 29.1

Before moving on, here is a summary of what you should have learnt from this section:

- (i) Electrons, nuclei, atoms and molecules exhibit observable diffraction effects.
- (ii) The diffraction patterns show that the wavelength associated with these forms of radiation is given by the de Broglie formula, $\lambda = h/p$.
- (iii) The lack of observable diffraction effects for large objects does not mean that de Broglie's formula cannot be applied to these objects. The wavelength expected on the basis of the formula would be too small to be noticed.
- (iv) The mathematics governing light propagation and that governing the motion of bodies have been shown to be equivalent. Thus not only light (as was seen in Unit 28) but all forms of energy are propagated in accordance with wave theory.
- (v) If the position of a body is localized in space, the mathematical wave associated with it is also localized in space—it is a wave packet. Such a wave packet does not have a precise wavelength; it can only be associated with a distribution of wavelengths.

29.2 How Energy and Momentum are Transferred in Interactions

Having seen how radiations are propagated in space and time, we now turn to a totally different question—how do radiations behave when they reach their destination and interact with matter?

29.2.1 The interactions of macroscopic objects, nuclei and electrons

We need say little about macroscopic objects such as pucks on air tables; these have already been dealt with in Units 3 and 4. There you saw that collisions between bodies involve sudden exchanges of energy and momentum and these exchanges are governed by the laws of energy and momentum conservation. The same is true also of collisions between nuclei and between electrons. For example, in Figure 17 an electron enters a bubble-chamber and strikes another electron belonging to an atom of the liquid. The second electron is hit sufficiently hard to be ejected from its parent atom and it then goes on to make its own trail of bubbles. Measurements on the tracks of the recoiling electrons show that momentum and energy are conserved in these collisions just as in the collisions of large objects. (You will learn more about bubble-chamber tracks and the measurements made on them in Unit 32.)

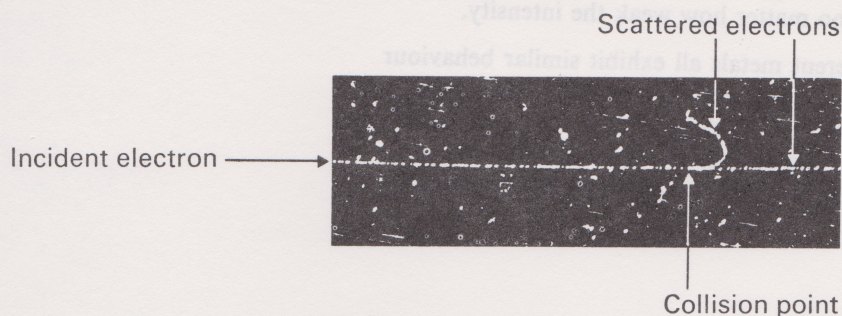


Figure 17 A bubble-chamber photograph showing an electron entering from the left and leaving a trail of bubbles. It strikes one of the electrons belonging to an atom of the liquid. The atomic electron is violently knocked away from its original atom and forms a track of its own.

Macroscopic objects, molecules, atoms, nuclei, and electrons are all found to interact in the same way—their interactions can all be explained in terms of a sudden exchange of energy and momentum according to the principles of energy and momentum conservation.

29.2.2 The interaction of electromagnetic radiations

To help you understand how light interacts we describe two experiments. The first involves the *photo-electric effect*.

photo-electric effect

In section 2.5.3 of Unit 2, you learnt that when an electron leaves a piece of metal, the metal acquires a net positive electric charge (the result of the electron taking away a negative charge); this gives rise to a force of attraction which drags the electron back. If the metal is heated, however, the energies of the electrons are increased and some now acquire the necessary 'escape velocity'.

Heating the metal is only one way of ejecting electrons. It is found that electrons can also be emitted when light is shone upon the surface.

What must the electrons be gaining from the light?

The escaping electrons must be absorbing energy from the incident radiation. Electron emission of this type is called the photo-electric effect. One of its useful features is that it allows light to be converted into a flow of electric charge (i.e. a current) and this property is made the basis of many types of instrument, e.g. a photographic exposure meter, a photomultiplier tube (as used in the TV programmes of Units 2 and 3), a colorimeter (like the one in your Home Experiment Kit), etc.

Here are some basic experimental observations on the photo-electric effect—some of them you will see demonstrated on television. They are observations we want *you* to interpret.

- (i) The number of electrons emitted per second by light of a given frequency is directly proportional to the intensity of the light.
- (ii) The electrons are emitted with a range of kinetic energies extending from zero up to some maximum value.
- (iii) This maximum energy of the emitted electrons is independent of the intensity of the light; it depends instead upon the frequency of the light as shown in Figure 18.
- (iv) Below the frequency, f_0 (Fig. 18), no electrons are emitted, regardless of the intensity of the light. Above f_0 electrons are emitted immediately light arrives at the surface, no matter how weak the intensity.
- (v) Electrons emitted from different metals all exhibit similar behaviour to that shown in Figure 18, except that the threshold frequency, f_0 , varies from one metal to another. The slope of the line is independent of the nature of the metal, and in fact is found to be equal to h , Planck's Constant—the same constant as appears in the formula for the de Broglie wavelength.

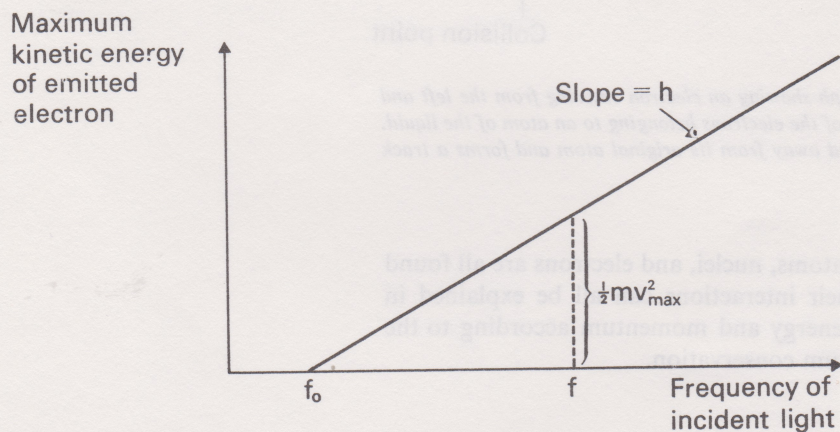


Figure 18 The maximum kinetic energy of electrons emitted from a metal surface by incident light of varying frequency.

On the basis of this experimental evidence, which of these models most satisfactorily describes the interaction of light with the electrons in the metal?

- (A) Light energy falls on the metal plate in small localized packets. Individual electrons absorb these energy packets and are emitted.

The energy in each packet is proportional to the frequency of the light.

(B) As (A) except that the energy in each packet is proportional to the intensity of the light.

(C) Light energy falls diffusely on the whole area of the metal surface. Certain electrons gather the energy from a comparatively wide area and, when they have sufficient, they escape.

Choose your model before reading further.

Model C offers no explanation of the cut-off frequency f_0 noted under observation (iv). Likewise model B is inconsistent with (i), (iii) and (iv). (For example (iii) specifically says that the maximum energy of the electrons is independent of the intensity.) Model A fits the data. According to this model the photo-electric effect is explained as follows:

Light transfers energy to individual electrons in localized packets. These packets are called *photons*. (You were first introduced to these in Unit 6.) *The intensity of the light governs how many photons arrive on a given area per second.* The electrons, on being struck by the photons, gain energy. Some of this is then expended by the electrons in escaping from the surface, the remainder appearing as the electron's kinetic energy. The least tightly bound electrons lose least energy in escaping and so emerge with the maximum kinetic energy. As we have said, the slope of the graph in Figure 18 is h , so we have the relation

$$h = \frac{\frac{1}{2}mv_{\max}^2}{(f-f_0)}$$

$$hf - hf_0 = \frac{1}{2}mv_{\max}^2$$

$$\therefore hf = hf_0 + \frac{1}{2}mv_{\max}^2 \dots\dots\dots(5)$$

where $\frac{1}{2}mv_{\max}^2$ is the maximum kinetic energy of electrons liberated by light of frequency f . From this expression it is concluded that *the energy of the incident photon is hf .* This energy is transferred to the electron on impact. For the least tightly bound electron a part of the energy, hf_0 , is needed to overcome the attractive forces appropriate to the particular metal being considered, while the remainder ($hf - hf_0$) goes into kinetic energy. The value of f_0 will vary from metal to metal (Fig. 19) depending

intensity related to the number of photons

energy of a photon is hf

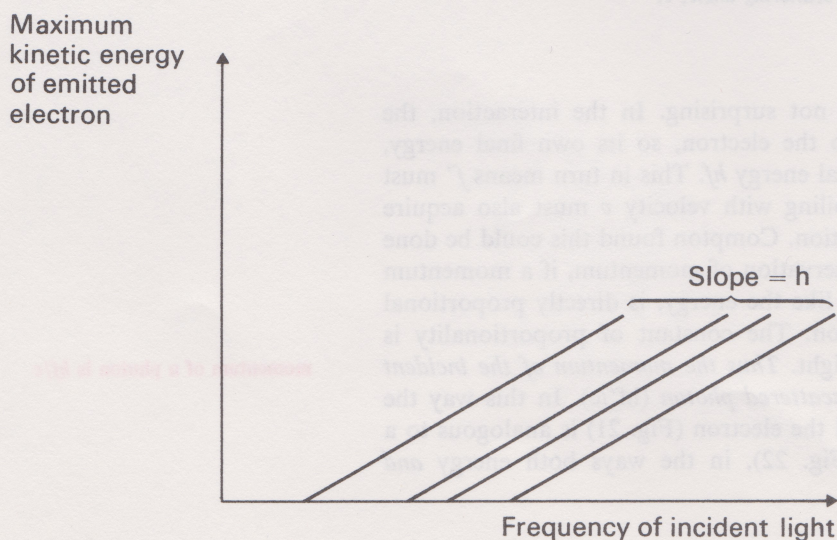


Figure 19 The same as Figure 18 for four different metal surfaces. Whereas the slopes are identical, the intercepts on the abscissa differ for each metal.

upon the strength of these forces. Electrons that are more tightly bound to their parent nuclei, i.e. are in the lower energy atomic states, require more than hf_0 in order to escape and so emerge with less than the maximum kinetic energy.

Now go back and check that all the experimental observations (i) to (v) are in agreement with this explanation.

As mentioned just now, you were first introduced to the idea of photons in the context of atomic spectra (Unit 6). It was found there that when electrons transferred from one atomic state to another the light was emitted in the form of discrete packets of energy. These photons were also assigned an energy hf . This assignment allowed the energy difference between the initial and final electron states to be determined from a knowledge of the frequency of the spectral line associated with the transition. Thus, whether light is being emitted or absorbed, its interaction with matter may be described by a theory involving the transfer of energy in the form of photons.

We turn now to the second of the two experiments concerned with the interaction of electromagnetic radiations. How is momentum exchanged in interactions between light and matter? (The fact that momentum is exchanged will be demonstrated in this Unit's TV programme.)

In 1924, the American physicist Compton studied the scattering of X-rays. (You will remember from Unit 2 that X-rays are a form of electromagnetic radiation.) In this experiment, it was found that when material is irradiated with a beam of X-rays of a single frequency, f , the radiation scattered from the electrons at any given angle, θ , has a reduced frequency f' (Fig. 20). This is called *Compton scattering*. For each angle of scattering, there is a characteristic value of f' . That the

Compton scattering

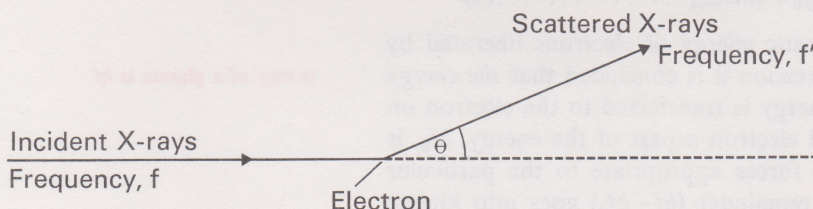


Figure 20 X-rays scattered from electrons have their frequency reduced, i.e. f' is less than f . The value of f' depends upon the scattering angle, θ .

frequency should be reduced is not surprising. In the interaction, the photon transfers energy $\frac{1}{2}mv^2$ to the electron, so its own final energy, hf' , must be smaller than its initial energy hf . This in turn means f' must be less than f . An electron recoiling with velocity v must also acquire momentum, mv , from the interaction. Compton found this could be done without violating the law of conservation of momentum, if a momentum is assigned to each photon that, like the energy, is directly proportional to the frequency of the radiation. The constant of proportionality is (h/c) where c is the velocity of light. Thus the momentum of the incident photon is (hf/c) and that of the scattered photon (hf'/c) . In this way the collision between the photon and the electron (Fig. 21) is analogous to a collision between two bodies (Fig. 22), in the ways both energy and momentum are exchanged.

momentum of a photon is hf/c

The equations expressing momentum and energy conservation for the two cases are given in Appendix 1 (Black).

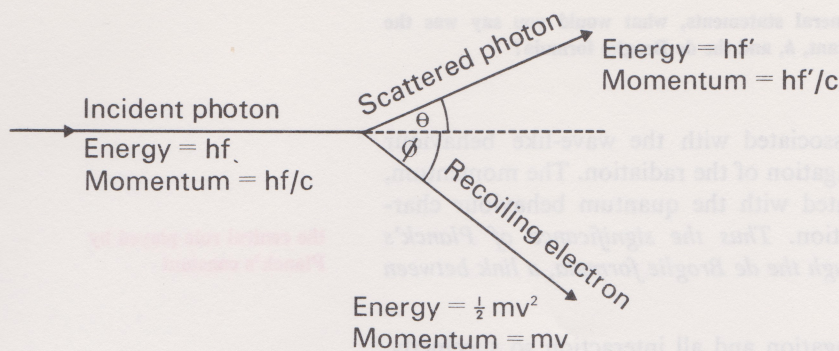


Figure 21 Diagram for illustrating the collision between a photon and an electron (Compton scattering).

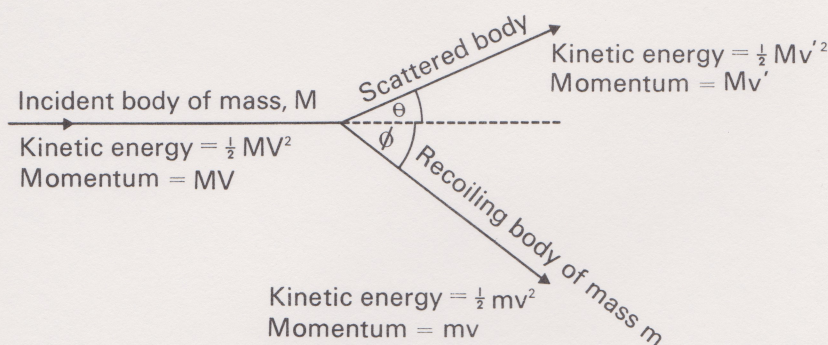


Figure 22 Diagram for illustrating the collision between two bodies.

The similarity between the behaviour of electromagnetic radiation and other types of radiation, such as beams of electrons and protons, can be shown to be even more striking than has been suggested so far. The wavelength of electromagnetic radiation in a vacuum is given by $\lambda = (c/f)$. Multiplying both the numerator and denominator of this ratio by h we get:

$$\lambda = \frac{hc}{hf} = h\left(\frac{c}{hf}\right)$$

But (hf/c) is the value of the momentum p of the photons. Therefore:

$$\lambda = h/p$$

This of course is the de Broglie formula again! This remarkable relation is therefore seen to apply to *all* forms of radiation—including light. In fact, photons are *so* similar to electrons, protons, neutrons etc. when they interact, that it becomes useful to introduce a new word, *quantum*. Thus photons, electrons, protons, etc., are all referred to as quanta.

quantum

We have now introduced you to two broad generalizations. Try to summarize them in two brief statements.

29.2.3 Summary so far

To our statement that all radiations propagate as waves, we are now in a position to add a second generalization:

THE INTERACTION OF RADIATION WITH MATTER MAY BE DESCRIBED BY THE THEORY OF QUANTA IN TERMS OF ENERGY AND MOMENTUM AND THEIR CONSERVATION PRINCIPLES.

all interactions governed by
quantum behaviour

In relation to these two general statements, what would you say was the significance of Planck's constant, h , and the de Broglie formula?

The wavelength, λ , is clearly associated with the wave-like behaviour characterizing the mode of propagation of the radiation. The momentum, p , on the other hand is associated with the quantum behaviour characterizing the mode of interaction. *Thus the significance of Planck's constant, h , is that it forges, through the de Broglie formula, a link between the two types of behaviour.*

To be able to describe all propagation and all interaction so succinctly, you must surely agree, is immensely satisfying. One of the primary aims of any scientist is so to order his observations that he can describe all phenomena in terms of only a few basic principles; quantum theory is certainly one of the most powerful unifying principles. But it is more than that; as you will see in the remaining sections, it has deep philosophical implications too.

the central role played by Planck's constant

An Aside

You may have been puzzled by our overall approach to the subject so far—this idea of taking a very general type of question involving some aspect of behaviour, and applying it to everything. You may be wondering why did we not get straight to the point and answer simple direct questions like: ‘What is light made of—waves or particles?’; ‘What are electrons—waves or particles?’. These latter questions were indeed the ones that early investigators of quantum theory tried to answer—and it was this that largely contributed to the confusion of the period. Quantum theory does not answer questions such as these, instead it answers questions of the type: ‘Given a particular situation, how will so-and-so behave?’ *It is a theory that describes behaviour, not attributes.*

quantum theory is concerned with behaviour, not attributes

This is no peculiarity or limitation of quantum theory alone; it can be argued that *all* science is concerned only with describing how things behave and not with what things are. If, for example, we say, ‘This object is blue’, what does such a statement mean—is it a statement of some attribute of the object? The object may certainly appear blue in normal daylight, but when examined in red light it will absorb all the light and appear black, not blue. If it is heated, it will glow red. Why, then, do we say the object is blue and not black or red? The reason is that we have all come to accept a convention, namely that the statement, ‘This object is blue’, is a convenient way of saying something like, ‘When this object is irradiated with white light, it reflects only the blue’. Note that this last statement is cast in the form of ‘Given a particular situation, this is how so-and-so behaves’. As long as everyone understands what convention is being used, it is, of course, perfectly all right to continue to make statements like, ‘This object is blue’; investigators working under different conditions (by the light of red lamps or in hot environments) will not think that they have come across a contradiction. But in the historical development of quantum theory, ‘contradictions’ of this type abounded. Sometimes electrons were thought to be waves, sometimes particles. How could they be both waves and particles? (In our analogy, how could the object be blue *and* black *and* red?) The resolution of these conflicts lay in a closer examination of the circumstances under which the behaviour of the electrons and other forms of radiation were being studied in the various experiments. Gradually it came to be realized that these circumstances were of two kinds. Sometimes the experiment was asking a question of the kind, ‘Given that a certain type of radiation is passing through an experimental arrangement of apparatus, where will it go (i.e. how will it behave)?’; at other times the question was of the general form, ‘Given that a certain type of radiation encounters matter, how will it interact?’.

In our presentation of the subject we have tried to heed the lessons of the past and have avoided the pitfalls of the conveniently short, but potentially ambiguous, question, ‘What is this type of radiation?’, and instead have directed you from the beginning to the right types of behavioural question.

Before rejoining the main text there is one further point we would like to make. In what follows, there will be a great deal of discussion about the behaviour of individual quanta—for instance, the diffraction of an electron by a slit. Now, it has already been pointed out that the diffraction of macroscopic bodies is negligible. There is a temptation, therefore, for you to feel that the following discussion has little relevance to an understanding of your everyday environment. Why should you have to bother with it?

Well, it is certainly true that quantum effects *are* negligible when many atoms combine to form a large object and this large object behaves as a single entity (as was the case with the billiard ball). But many aspects of

the relevance of quantum theory to everyday life

everyday life, although involving large numbers of atoms, reflect the characteristics of the atoms behaving *individually*, rather than collectively.

Take, for example, the properties of the elements. It is possible to imagine a world in which every atom is different from every other atom and, moreover, the properties of each atom varies with time (the motion of the electrons in the atom might perhaps ‘run down’ as the atom ‘wears out’). The world you live in, however, is simply not like that. It appears that only certain restricted types of atom (i.e. elements) can exist and every atom must belong to one or other of these types. Moreover, atoms of iron, say, retain exactly the same behavioural characteristics over any length of time—one cannot speak of ‘fresh young’ iron atoms or ‘old worn-out’ ones. As you will see in Unit 30, the constancy of their behaviour is a direct result of the application of quantum theory to the structure of atoms. It is true that a cannon ball may show little signs of being diffracted as it leaves the muzzle, but the physical and chemical properties of *iron itself* depend directly on the nature of the individual iron atoms—and their characteristics are governed by quantum behaviour.

Or, as another example, one may consider the light being emitted from a heated filament bulb. You can, of course, see the light from such a heated filament, and quantum theory can explain this. The theory that preceded quantum theory, however, could not explain this simple fact. According to this earlier theory all the light would be emitted in the ultra-violet region and you would not be able to see it! We do not wish to go into an explanation of this—we merely state it as a fact that the wavelength of the light by which you are reading this text (whether it is light from a filament bulb, or a neon strip light, or daylight) is governed by quantum behaviour and the value of Planck’s constant. Once again this is because the characteristic of interest—the wavelength of the light—does not depend on how many atoms are involved, but on the nature of individual atomic collisions.

In these and many other ways, quantum theory directly impinges on everyday life and moulds the environment—the discussion that follows is far from being an academic exercise.

29.3 Probability waves

In the photo-electric effect, the number of electrons released from an illuminated metal plate was found to be proportional to the intensity of the light shining on it. If each electron is ejected by a collision with a photon, the intensity of the light falling on a given surface must be proportional to the number of photons arriving on that surface. Now suppose light passes through a narrow slit as in Figure 23. In this diagram, as in all subsequent diagrams showing arrangements of slits, it is to be understood that the long dimension of the slit is at right-angles to the plane of the paper; the gap shown represents the width of the slit.

Earlier it was shown that for this experimental arrangement the form of the intensity distribution is that of Figure 5. From this distribution, it can be seen that, for example, the intensity of the central maximum is about 20 times that of the first diffraction maximum. It therefore follows that 20 times the number of photons arrive on the screen in a small region opposite the slit (position A of Fig. 23) as will arrive on an area of equal size situated at the first diffraction peak (position B). Thus, the wave behaviour determines how many photons arrive on each part of the screen (the quantum behaviour of course determines how the photons interact once they arrive).

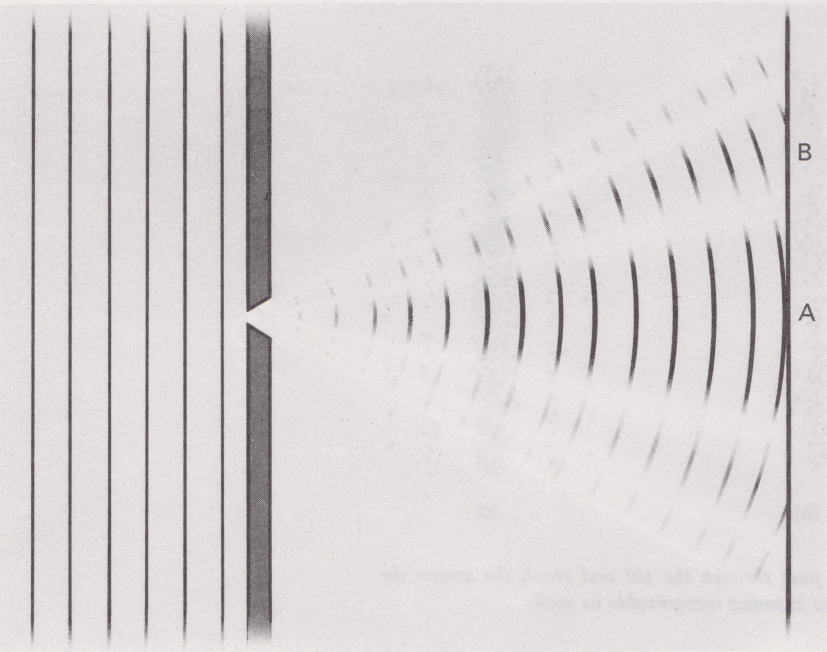


Figure 23 Diagram illustrating the diffraction of light at a slit. There are regions of maximum intensity on the screen at positions such as A and B.

So far so good. But what if the slit is opened and closed very rapidly so that the average energy transmitted in the time it is open is less than the energy of a single photon. Does only a fraction of a photon arrive at the screen? We appeal once again to experiment. It is found that when light of very low intensity is allowed to pass through an opening for a tiny

fraction of a second (in one particular version of the experiment the time was one nano second) either a photon with the whole energy, hf , arrives at the screen or none at all.

Given then that only a single photon's worth of light arrives at the screen, what happens to the intensity distribution? Does the photon smear itself out to form a diffraction pattern?

No. The photon does *not* spread itself out—that much is once again established by experiment. When a single photon arrives, it strikes one, and only one, position on the screen. Whereabouts on the screen it will strike cannot be predicted.

Are there any regions of the screen to which you might guess the photon would *not* go?

All that can be known in advance is that it will arrive at a point on the screen where the diffraction pattern (*calculated* from its wavelength and the size of the slit) would be non-zero, i.e. the photon is liable to arrive anywhere on the screen other than at positions lying on a calculated diffraction minimum. (Note the stress on the word 'calculated'. When dealing with a single photon there is no *observable* diffraction pattern; it can only be *calculated* from the known wavelength and the size of the slit.)

Likewise a second photon will go to some position on the calculated diffraction pattern. Subsequent photons do the same, and gradually with time the form of a diffraction pattern materializes as in Figure 24.

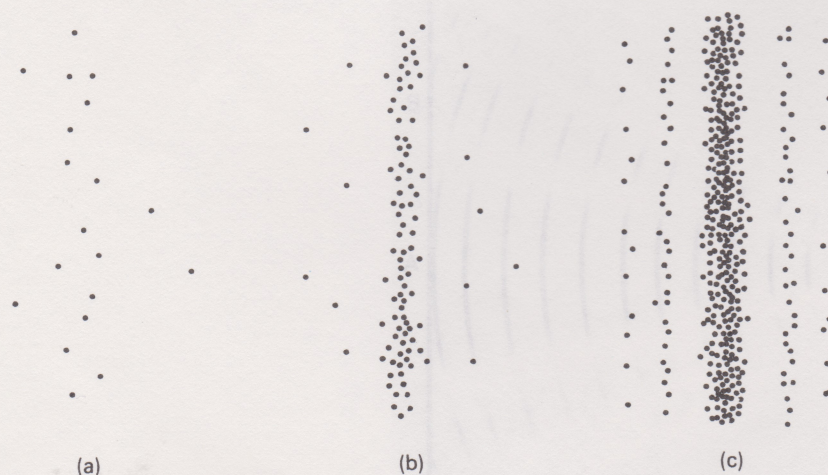


Figure 24 As more and more photons pass through the slit and reach the screen the diffraction pattern gradually builds up and becomes recognizable as such.

Figure 25 shows the same kind of effect with electrons. These photographs were taken with an electron microscope. They show the gradual build up of a diffraction pattern for electrons diffracted at the edges of a small hole.

Thus, the calculated diffraction pattern describes the *probability* of a photon arriving at various parts of the screen. Where exactly any individual photon will go cannot be predicted—we can only hope, with the help of wave mathematics, to calculate the odds on it going to one place rather than another.

the calculation of probabilities

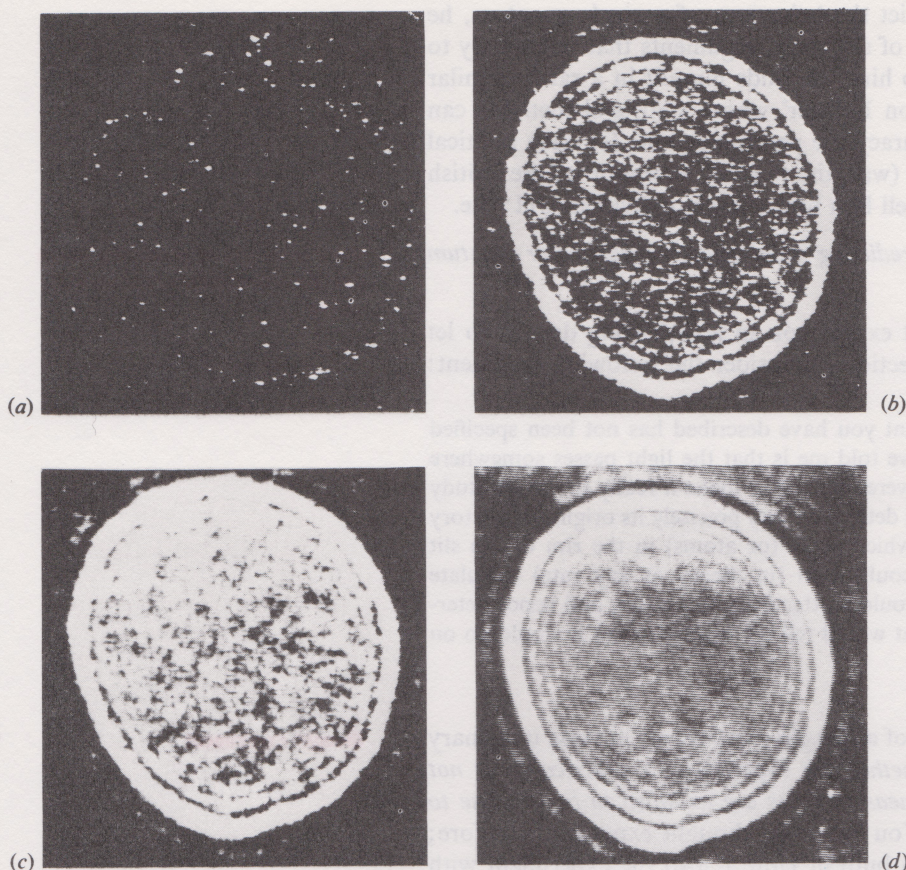


Figure 25 Actual photographs taken with a very low intensity electron beam in an electron microscope. Each dot is an image formed by a single electron. The exposure times were (a) 1/25th second, (b) 10 seconds, (c) 1 minute, and (d) 2 minutes. Note how the form of the diffraction pattern becomes clearer as the number of electrons in the pattern increases. (Magnification $\times 125\,000$.)

What are the chances of a given photon going to region A of Figure 23, compared with its chances of going to region B?

To talk of being able to predict only the odds on the outcome of an experiment may well strike you as very strange. After all, you have probably been told in the past that physics is an exact science. If a physicist repeats an experiment he has often performed before, he is expected to be able to specify in advance what the outcome will be. To a large extent, of course, this remains true; if he takes a quantity of gas and halves the volume, keeping the temperature constant, he knows this will double the pressure; if he doubles the temperature difference across the faces of a sheet of material, he knows this will double the heat transmitted through it, etc. In these, as in many other examples, you can no doubt think of for yourself, the outcome is known—at least to a high degree of precision. However, you should note that these examples deal only in gross features; they each involve the average effects produced by an enormous number of atoms—in one case, the momentum imparted by many gas atoms to the walls of a container averaged over a period of time; in the other, the energy transferred between large numbers of vibrating atoms as they jostle each other. In this respect, they are like the experiment in which a substantial amount of light is allowed to pass through the slit—in that case also the physicist is able to predict the outcome of the experiment, i.e. he knows in advance that, given enough photons, he will get a diffraction pattern of a certain size and intensity distribution, such as will allow him to predict, for example, that 20 times as many photons will arrive at region A as at region B (Fig. 23).

The intensity of the diffraction pattern in region A is 20 times greater than in region B. Thus, the number of photons arriving there is 20 times greater, and consequently the chance that any given photon will arrive there is 20 times greater.

But, when he is asked to predict the behaviour of a *single* quantum, he is unable to do it. In this kind of situation, arguments that relate only to statistical averages do not help him. He finds himself in a rather similar position to that of the 'Opinion Pollster' whose statistical methods can forecast with considerable accuracy the percentage vote for each political party averaged over the nation (with the possible exception of the British election of 1970!), but do not tell him how a given individual will vote.

We repeat: *there is no way of predicting where exactly a particular quantum will arrive.*

there is no way of predicting where exactly a particular quantum will arrive

That is a statement we do not expect you to accept lying down! So let us try and anticipate your objections. Consider the following argument:

The nature of the experiment you have described has not been specified sufficiently well; all you have told me is that the light passes somewhere through the slit. Now, if I were allowed to make a really thorough study of the initial photon, I could determine very precisely its original trajectory and could then work out which atom (or atoms) in the rim of the slit it would interact with. I could then (in theory at any rate) calculate precisely how the photon would scatter from this atom, and hence determine its final direction. That would tell me exactly where it would go on the screen.

This incidentally is an example of a *thought experiment*; in these imaginary experiments the question of whether the experiment is practicable or not is ignored. All equipment and measurements are assumed to be as close to the ideal as can be imagined. You have met thought experiments before; remember the car with its flash bulb in Unit 3, also the experiment with the train to demonstrate the failure of the idea of simultaneity as applied to events separated by a distance. Thought experiments have always been regarded as particularly useful in clarifying the meaning and significance of quantum theory.

thought experiment

At this point we throw down a challenge to the really bright student!

Think back over the material that has so far been presented in this Unit, and see if you can formulate two or three convincing arguments to show what is wrong with the assumptions or line of reasoning in this thought experiment.

If you think that this is too difficult, most of the Course Team would probably agree with you! But don't give up just yet. Try the three clues below and make a genuine attempt to develop the line of reasoning suggested in each before reading on.

Clue 1

The precise study of the collision between the photon and the atom would presumably involve a detailed knowledge of the struck atom. Have you been told anywhere that the diffraction of light through a slit in a barrier depends on the nature of the material of the barrier?

Clue 2

Is it possible, even in principle, to determine precisely the trajectory of the initial photon? Try designing an experimental arrangement for doing this.

Clue 3

Suppose you opened up a second slit in the barrier very close to the first—what effect would this have on the calculated intensity distribution? Can you reconcile this change of probabilities with a photon whose final direction can be precisely specified by the way it scatters on the rim of only one of the holes?

In a simple theory of a precisely determined photon-atom collision, it is difficult to see why there should be no possibility at all of the photons scattering at certain angles (to the diffraction minima). Moreover, it is natural to assume that such directions of zero probability would in any case depend in some way on the atom involved in the collision—how

heavy it is, how it is oriented and how strongly it is bound to neighbouring atoms. Yet, if you remember, the directions of the diffraction minima depend *only* on the wavelength of the radiation and the width of the slit, not on the nature of the material surrounding the slit.

Secondly, it is all very well saying that the original trajectory is to be precisely determined, but how could one do this, even in principle? One might think, for example, of placing a tiny hole, H, in front of the source of light as in Figure 26. By bringing up H very very close to S, and by making it exceedingly small, one could learn precisely where the photon was as it passed through S (Fig. 27). But in order to know which atom was hit on the rim of S, H would have to be so very small, and the consequent diffraction it causes to the emergent light so large, that the photon could be moving in almost any direction as it emerged from H. Not knowing the direction of the photon makes it impossible, of course, to work out the details of its collision with the atom.

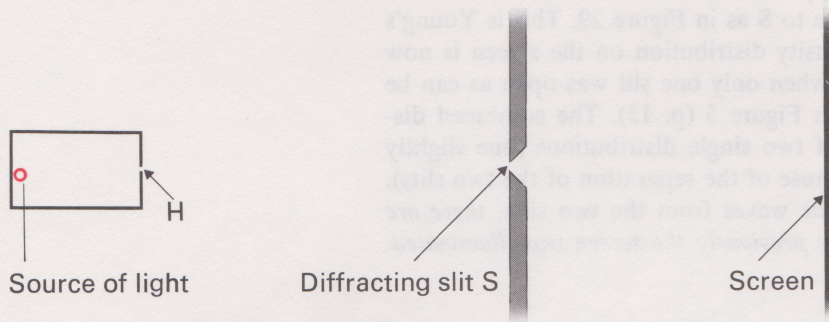


Figure 26 A hole, H, introduced in front of the source in an attempt to produce a well-defined beam of light.

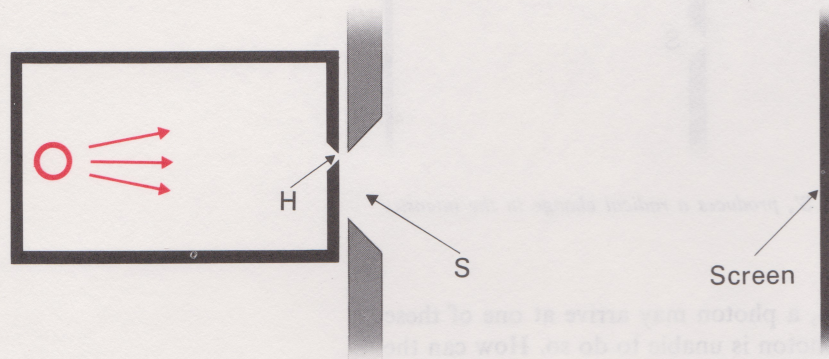


Figure 27 The hole, H, is made very small and is brought up close to the slit S. In this manner the position of the photon at S is well defined—but not its direction.

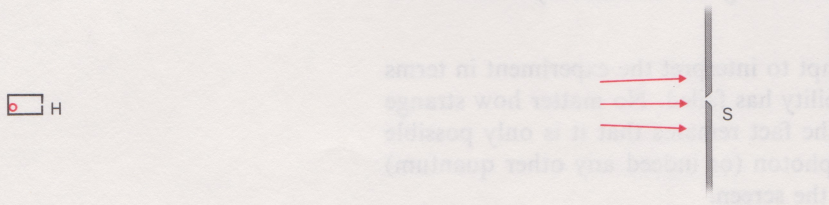


Figure 28 The hole, H, is removed a long way from S. In this arrangement the direction of the photon at S is well defined—but not its position.

In order to fix the direction of the photon, the source and hole H would have to be taken a very long way from S (Fig. 28). Now, because the angle subtended by the width of S at the source is so small, the fact that

the photon is known to pass somewhere between the edges of S defines its direction very satisfactorily.

Is it possible to work out the details of the collision now?

Unfortunately, in order to get this precise knowledge of the initial direction of the photon, our knowledge of the position of the photon as it passes through S has had to be sacrificed. It is now not known whether the atom is being struck a glancing blow or a direct hit, or indeed even which atom is being struck.

A discussion of this kind soon reveals that it is quite impossible, even in principle, to specify a precisely determined trajectory; an argument that assumes such a determination must therefore be fallacious.*

A third and final refutation: suppose a second slit, S', identical and parallel to the first, is opened close to S as in Figure 29. This is Young's double-slit arrangement. The intensity distribution on the screen is now radically different to what it was when only one slit was open as can be seen by comparing Figure 30 with Figure 5 (p. 13). The combined distribution is not simply the sum of two single distributions (one slightly displaced relative to the other because of the separation of the two slits). Because of interference between the waves from the two slits, *there are now regions of zero intensity where previously the screen was illuminated.*

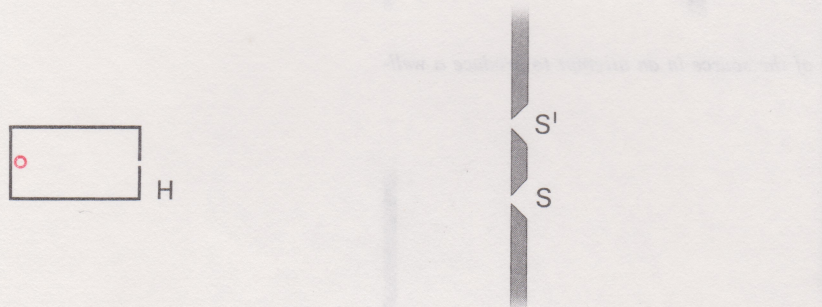


Figure 29 The opening of a second slit, S', produces a radical change in the intensity distribution at the screen on the right.

This means that if one slit is open, a photon may arrive at one of these regions, but if *both* are open the photon is unable to do so. How can the opening of the second slit (a slit the photon presumably does not pass through) affect the outcome of the photon-atom collision in the first slit in a way that makes it no longer possible for the photon to go to one of these regions? Clearly anything resembling conventional dynamics could not possibly give such a result.

Thus, on several counts, the attempt to interpret the experiment in terms other than those involving probability has failed. No matter how strange the idea appears to you at first, the fact remains that it is only possible to predict the probability that a photon (or indeed any other quantum) will arrive at any given region on the screen.

* You could of course argue that the diffraction effects at hole H would not be a problem if it were possible to study the collision of the photon with the atoms in the rim of H, before studying its subsequent collision with the atoms in the rim of S. We leave you to spot what is wrong with that.

You may like to amuse yourself by dreaming up other ways of trying to define a precise trajectory for the photon. (If at any stage you think you have succeeded, don't get too excited—you may have overlooked something!)

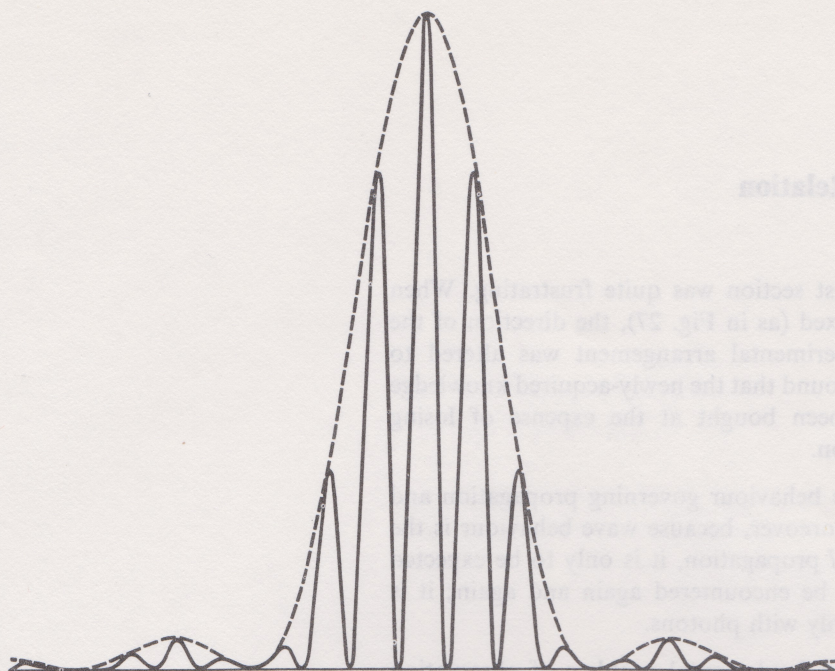


Figure 30 The intensity distribution on the screen resulting from light passing through Young's double-slit arrangement.

As was said earlier, in order to calculate this probability, the same type of mathematics must be used as is appropriate in describing the progress of, for example, ripples on the surface of water or sound in a gas; it is the mathematics of waves. A quantity can be defined to represent the 'amplitude' of the 'probability wave'. This behaves mathematically in the same way as the amplitudes of water or sound waves, inasmuch as two or more probability wave amplitudes can be added to give constructive or destructive interference. The square of the amplitude of a water or sound wave is a measure of the energy of that wave; the square of the amplitude of the probability wave at any given position is a measure of the probability of finding the quantum at that position.

But what, you may ask, is 'waving' in these probability waves? What is the medium? Here the analogy with water and sound waves breaks down (you have been warned before of the dangers of taking analogies too far!). The latter require a medium—probability waves do not. The similarity is a similarity in mathematics only. *The probability waves say nothing about the state of any medium; instead they tell us of the state of our knowledge.*

Perhaps you find an abstract concept such as this unpalatable. But why should you? Presumably you did not flinch when in earlier Units we discussed the Earth's orbit around the Sun—but what is an orbit? If you went out into space you would see no line drawn on anything. An orbit is an abstract concept useful for describing our knowledge of successive positions of the Earth. If you are happy to use a mathematical line in space as a convenient summary of our knowledge of the Earth's motion, why not mathematical waves?

probability waves are not associated with any medium

29.4 Heisenberg's Uncertainty Relation

The situation described in the last section was quite frustrating. When the position of the photon was fixed (as in Fig. 27), the direction of the photon was not. When the experimental arrangement was altered to remedy this (as in Fig. 28), it was found that the newly-acquired knowledge of the photon's direction had been bought at the expense of losing knowledge of the photon's position.

The trouble stems from the wave behaviour governing propagation and its probabilistic interpretation. Moreover, because wave behaviour is the underlying principle governing *all* propagation, it is only to be expected that this kind of frustration will be encountered again and again; it is not some peculiarity associated only with photons.

In this section, we shall study this fundamental problem of observation in greater depth. We shall investigate to what extent it is possible simultaneously to measure two properties of a quantum: its position and its momentum. For this purpose, we shall again use a slit. The act of measurement is considered to take place at the instant the quantum arrives at the slit. (Although we are sticking to our example of diffraction at a slit, you should realize that what is being discussed has universal significance; we could just as well have chosen some other experimental arrangement with which to illustrate the point we wish to make.)

In Figure 31, a drawing of the experimental arrangement is combined with one of the intensity distribution to remind you of the appearance of the diffraction pattern on the screen when the slit is illuminated with monochromatic radiation from a distant source. The y axis has been drawn at right angles to the length of the slit. (Remember that the length of the slit is taken to be perpendicular to the plane of the paper.)

At the instant at which the quantum arrives at the slit, its position along the y axis is uncertain; it is only known that it must lie somewhere between the extreme edges of the slit. If the width of the slit is d , then the uncertainty in the position of the quantum along the y axis—call it Δy —at the instant it arrives at the slit is given by: $\Delta y = d$.

At this instant, the quantum is 'diffracted', i.e. receives a sideways impulse resulting in it subsequently arriving at the far screen at a position L, which may be other than position O directly opposite the slit from the source (Fig. 31). How large the impulse will be, no one can say. The photon must therefore be regarded as having an uncertain momentum in the y direction as well as an uncertain position. (Let us emphasize that we are here talking of uncertainty of both position and momentum at the moment the quantum arrives at the slit. Do not be confused into thinking we are referring in any way to uncertainty of position on the screen; the spread of positions of quanta on the screen will only be used as a convenient measure of the uncertainty of the y component of momentum at the slit.)

If the point L is such that the direction SL makes an angle ϕ to the original direction of the light, then the y component of momentum would be $p \sin \phi$ (Fig. 32), where p is the total momentum of the quantum (both before and after 'diffraction'). But, as we have said, at the time of the measurement there is no way of telling what the value of ϕ will be. All we can do is take a look at the spread of the calculated diffraction pattern and decide on some average 'typical' value for the angle.

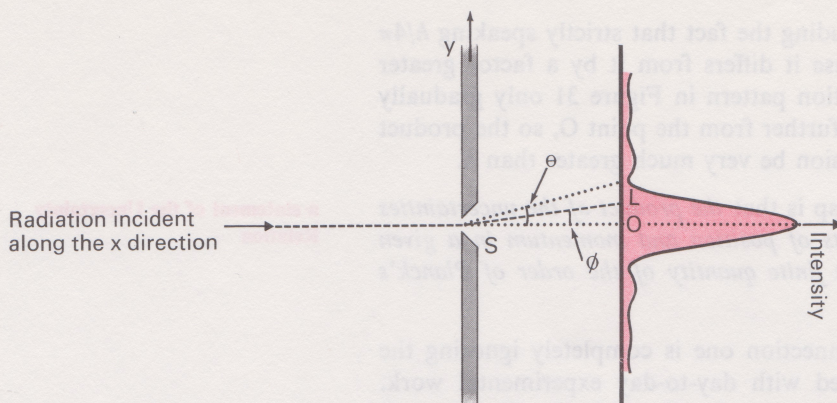


Figure 31 A drawing combining the experimental arrangement and the intensity distribution on the screen appropriate to radiation of a single wavelength.

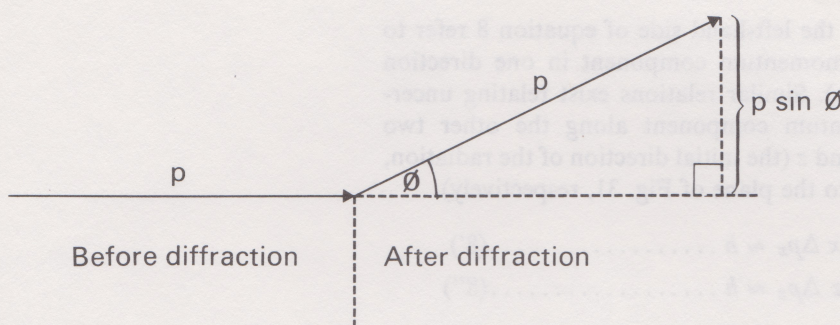


Figure 32 After diffraction, the quantum acquires a sideways momentum $P \sin \phi$.

What would be a typical value of ϕ ? One possibility is to say that most of the radiation would be diffracted to somewhere within the central intensity peak, i.e. ϕ will typically extend to the first diffraction minimum. The direction of this minimum makes an angle θ to the original direction. The ‘typical’ sideways momentum is therefore $p \sin \theta$, and this we take as a measure of the uncertainty in sideways momentum, Δp_y .

$$\therefore \Delta p_y = p \sin \theta \dots\dots\dots(6)$$

As shown in Unit 28, there is a relation connecting θ , λ , and d :

$$\sin \theta = \lambda/d$$

Substituting this value for $\sin \theta$ in equation 6 we have:

$$\Delta p_y = p(\lambda/d)$$

If Δy and Δp_y are multiplied together we get

$$\begin{aligned} \Delta y \Delta p_y &= d.p(\lambda/d) \\ \Delta y \Delta p_y &= p\lambda \dots\dots\dots(7) \end{aligned}$$

But we have the de Broglie relation connecting λ and p , and from this we can substitute (h/p) for λ in equation 7:

$$\begin{aligned} \Delta y \Delta p_y &= p(h/p) \\ \Delta y \Delta p_y &\approx h \dots\dots\dots(8) \end{aligned}$$

Heisenberg’s Uncertainty Relation

This is Heisenberg’s Uncertainty Relation—or at least one form of it. Note that the ‘equals’ sign has been changed in the last line to another sign meaning ‘of the order of’ (i.e. ‘roughly within a factor of ten of’). This is because of the rather cavalier way in which a typical angle of diffraction was defined. In other texts you will find h replaced by $(h/2\pi)$ or $(h/4\pi)$; it all depends on how exactly one chooses to define the quantities on the left-hand side of equation 8. We shall not concern

ourselves with such details (including the fact that strictly speaking $h/4\pi$ is not 'of the order of' h because it differs from it by a factor greater than ten!). After all, the diffraction pattern in Figure 31 only gradually falls off in intensity as one goes further from the point O, so the product of the uncertainties can on occasion be very much greater than h .

The crucial point you are to grasp is that *the product of the uncertainties in the simultaneous measurements of position and momentum in a given direction is equal to a small but finite quantity of the order of Planck's constant.*

a statement of the Uncertainty Relation

We remind you that in this connection one is completely ignoring the usual sources of error associated with day-to-day experimental work. Throughout this Unit we are concerned only with the fundamental principles of measurement. These show that *even with a perfect idealized experiment*, the measurements of position and momentum are subject to limitations imposed by the uncertainty relation.

Note also that the quantities on the left-hand side of equation 8 refer to measurements of position and momentum component in one direction only (in this case the y direction). Similar relations exist relating uncertainties in position and momentum component along the other two directions at right angles, viz. x and z (the initial direction of the radiation, and the direction perpendicular to the plane of Fig. 31, respectively).

$$\Delta x \Delta p_x \approx h \dots\dots\dots(8')$$

$$\Delta z \Delta p_z \approx h \dots\dots\dots(8'')$$

In essence, the uncertainty relation does not tell us anything more than is already contained in the material presented in the previous sections of this Unit; it is certainly not a new postulate. It arises directly out of the concept of probability waves. The idea of probability waves is sufficient in itself to tell us that a slit of small width, used to fix precisely the position of a quantum, will produce large diffraction effects and hence large uncertainties in the sideways momentum of the quantum. It also tells us that one can alternatively have very little diffraction if one is prepared to open up the slit wide. The importance of the uncertainty relation is that it sums up in a particularly cogent fashion what one can or cannot hope to measure in any experimental situation—it is a powerful 'rule of thumb'.

In the footnote on p. 36, we made a provocative statement. You were given a free hand to devise alternative ways of defining a precise trajectory for the initial quantum—and without waiting to learn what ideas you came up with, we summarily dismissed them all as failures. This may well have stung you into producing all sorts of designs just to prove we were being high-handed!

Can you now see why it was possible to make such a sweeping judgement?

In order to specify a trajectory exactly, you would have to be able to specify at a given instant both the exact position of the quantum and the exact values of the components of its momentum. Equation 8 says quite simply that you cannot do it. A precisely determined position (i.e. no uncertainty in position and hence $\Delta y=0$) means that in order to keep the left-hand side of equation 8 from going to zero, Δp_y would have to be infinite, i.e. there would be no knowledge of the momentum. Alternatively a precisely determined momentum (i.e. $\Delta p_y=0$) implies no knowledge of position (i.e. Δy is infinite). This is exactly the kind of dilemma encountered in the arrangements of Figure 27 and Figure 28, when these two extremes were approached. Between these extremes, there is a con-

tinuous span of intermediate possibilities such that some gain in the precision of one variable may be made at the expense of a loss of precision of the other.

Thus, although it is necessary to know the exact experimental arrangement if one is properly to evaluate from wave theory the probabilities of the various outcomes of the experiment, the uncertainty relation allows one to make strong generalized statements. It is indeed somewhat unfortunate that the relation carries the name 'uncertainty' for this word implies hesitancy, indecision, and a general state of confusion. The student of quantum theory may well feel that if the professional physicist is 'uncertain' as to what he is doing, there is little hope for the student! For this reason the word 'indeterminacy' is sometimes preferred instead of 'uncertainty'. Regardless of what it is called though, you will soon come to recognize that the relation is more than just a semi-quantitative expression of frustration—it is a positive tool for describing many features of Nature.

Perhaps you still feel it goes against common sense to believe that one is unable simultaneously to measure both the position and the momentum of an object to any desired precision given that the measuring apparatus is ideal. Until some such time as the material of this Unit finds its way into the primary school curriculum, it will continue to be regarded as against common sense, and students will spend hours trying to find a way round the uncertainty relation. It is something we all apparently have to get out of our systems at one time or another. Even Einstein devoted long periods to devising ingenious schemes. The diversion of designing experimental arrangements for getting round Heisenberg's relation can be likened in many ways to the old quest for the perpetual motion machine!

To give you an idea of the pitfalls lying in wait for those who would outwit Heisenberg, we analyse a few approaches in Appendix 2 (Black).

If you are still in any doubts about the validity of the uncertainty relation, you should read Appendix 2.

We shall here merely draw your attention to two points that emerge from the discussion in that appendix:

Firstly, the observer, in making an observation, always disturbs the system he is observing. He cannot play a passive role, i.e. obtain his information without actively disturbing the system. This in itself is nothing new; classical ideas of measurement also recognize that an observer interferes with the object under investigation. What *is* new in quantum theory is that the disturbance is unpredictable—the measurement cannot be suitably corrected to take account of the disturbance.

Secondly, the observer has to make a choice between different courses of action—*either* to concentrate on precision of momentum, *or* precision of position, *or* some kind of compromise. Having decided on a particular kind of measurement, the unpredictable disturbance associated with the act of taking that measurement denies the observer the opportunity of ever gaining the information he *could* have obtained had he taken one of the alternative courses of action. For example, he could consider the alternatives whereby he either uses a small slit to measure the position of a quantum precisely, or a large slit (so keeping the diffraction effects small) to measure the momentum precisely. If he decides to use the small slit, then the very act of measuring the position of the quantum produces a disturbance (a large diffraction) that destroys the information on the quantum's momentum that he *could* have obtained had a large slit been used instead.

This is the essence of *Heisenberg's Uncertainty Principle*. When an observer is faced with a range of alternative procedures for measuring the momentum and position of an object (the precision of these measurements being governed by the uncertainty relation), he can gain only the information appropriate to one of the alternatives.

Indeed, the principle extends further than just to the measurement of the two variables: momentum and position. There are other pairs of variables linked by similar uncertainty relations. Thus, for example, closely associated with the concept of momentum is that of angular momentum. If the momentum of an object is uncertain, then it is only to be expected that the value of its angular momentum about some centre will also be uncertain. An uncertainty relation exists connecting the uncertainty in angular momentum, ΔL , and the uncertainty in angular position, $\Delta\psi$:

$$\Delta L \Delta\psi \approx h \dots\dots\dots(9)$$

another uncertainty relation

There is one more uncertainty relation you should know about; it concerns uncertainty in energy. It is so important that we devote the next section to it.

29.4.1 The energy-time uncertainty relation

In the experiment where a slit was used to measure the position and momentum component of a quantum, *when* exactly was the measurement made?

You may immediately answer: At the instant the quantum arrived at the slit. But how would one estimate the instant of arrival?

Perhaps the most direct way is to fix a shutter onto the slit as in Figure 33(a). The slit could then be opened for only a limited time (Fig. 33(b)). In order for the y position and momentum of the quantum to be measured, the quantum has to arrive within the time interval for which the slit is open. This interval can be made as small as one likes. In this way, it is possible to fix exactly the instant at which the measurement is made.

However, this procedure has serious repercussions. If the slit is only open for a short time, then the radiation transmitted is no longer in the form of an infinitely long monochromatic wave train—it is in the form of a sharp wave packet. But this is quite contrary to our earlier assumption; the diffraction pattern shown in Figure 31 is derived on the basis of a *single* wavelength. If we are dealing with a wave packet, this is the same thing as saying our radiation has many different wavelengths; we are now talking of a *different* experiment.

We can look at it another way. If the instant at which the quantum arrives at the slit is known, it follows that, at that instant, it is known exactly where the quantum is along the incident direction, i.e. along the x axis (it is at the position S). The wave packet describing its position in the x direction is therefore a sharp spike with a spread, Δx , almost zero. But, according to the uncertainty relation for this direction, (equation 8') a small value of Δx implies a large uncertainty in the momentum in the x direction, i.e. in the incident momentum p . In the original version of the experiment (as in Fig. 31), it was assumed that p was known precisely. We now see that such precision could only be achieved with a permanently open slit (resulting in Δx becoming infinitely large and Δp zero).

To summarize: in previous sections we were concerned only with making a measurement of the position and momentum of the quantum in the y direction. For this purpose it was *assumed* that we were dealing with a quantum of known momentum, p . If, however, we wish to say something about the time at which the measurement is made, it is necessary to perform a *different* experiment—one involving, for example, the opening

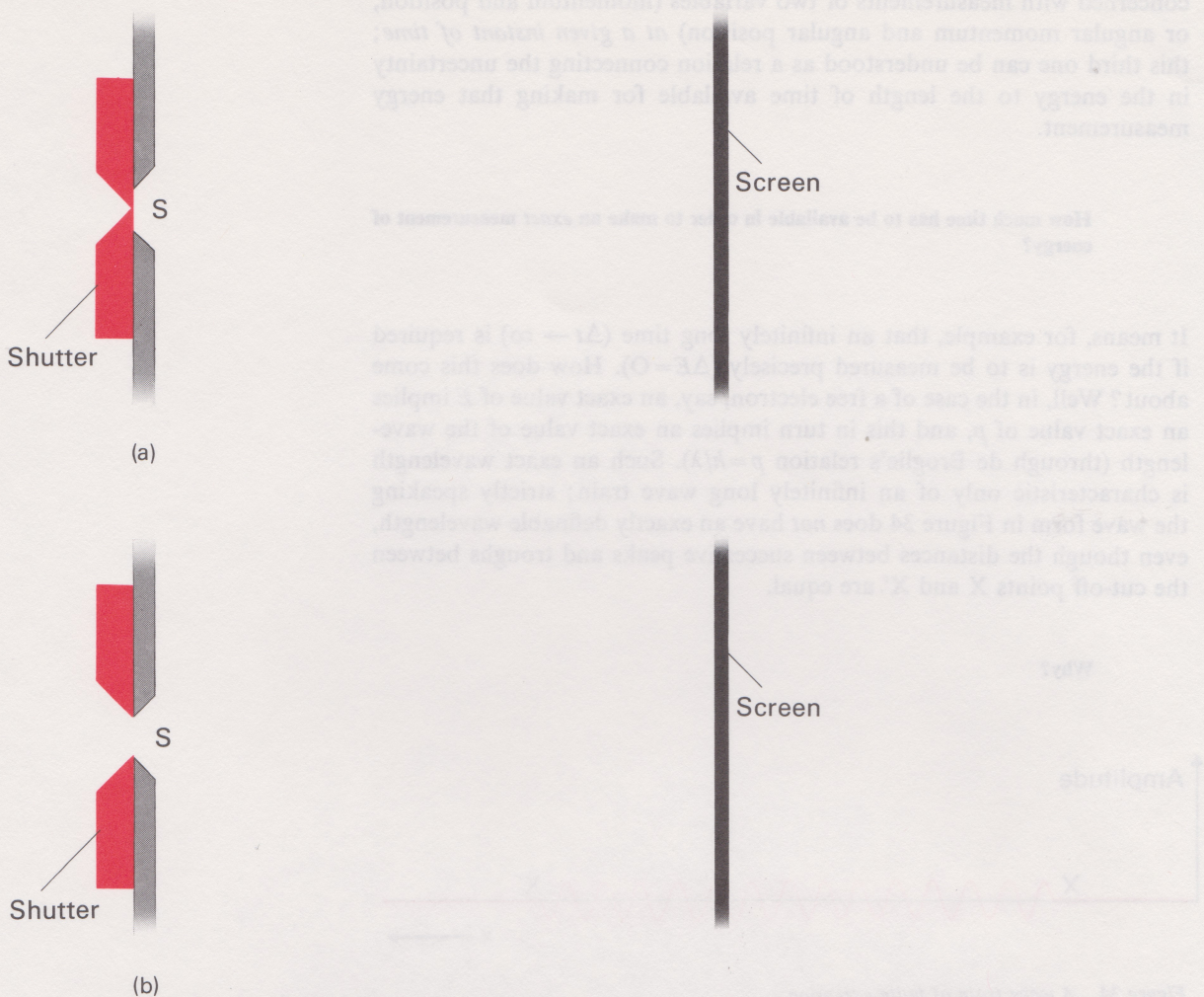


Figure 33
 (a) A shutter is now placed over the slit through which the radiation is to pass.
 (b) The shutter can be opened to admit the radiation.

and closing of the slit. This modification of the original experimental arrangement, while permitting us to say something about the time at which the measurement is made, affects our knowledge of p .

If our knowledge of p is uncertain, then this is the same as saying our knowledge of the energy, E , is uncertain; corresponding to Δp , there will be a ΔE . The value of ΔE (like Δp) will depend upon the time the slit is open. Let us call the latter Δt ; it is the time available for the measurement. There exists a relation connecting ΔE and Δt . This is a perfectly general relation and applies to all experimental arrangements (not just the slit arrangements we have been considering).

The uncertainty in the energy of a system, ΔE , and the time available for measurement, Δt , is given by:

$$\Delta E \Delta t \approx h \dots\dots\dots(10) \qquad \text{the energy-time uncertainty relation}$$

The derivation of this relation for the experimental situations we have been discussing is not difficult, and is carried out in Appendix 3 (Black).

You should note that this third form of uncertainty relation, although in appearance very similar to the previous ones (equations 8, 8', 8'' and 9), has a different kind of interpretation. The previous relations were all

concerned with measurements of two variables (momentum and position, or angular momentum and angular position) *at a given instant of time*; this third one can be understood as a relation connecting the uncertainty in the energy to the length of time available for making that energy measurement.

How much time has to be available in order to make an *exact* measurement of energy?

It means, for example, that an infinitely long time ($\Delta t \rightarrow \infty$) is required if the energy is to be measured precisely ($\Delta E = 0$). How does this come about? Well, in the case of a free electron, say, an exact value of E implies an exact value of p , and this in turn implies an exact value of the wavelength (through de Broglie's relation $p = h/\lambda$). Such an exact wavelength is characteristic only of an infinitely long wave train; strictly speaking the wave form in Figure 34 does *not* have an exactly definable wavelength, even though the distances between successive peaks and troughs between the cut-off points X and X' are equal.

Why?

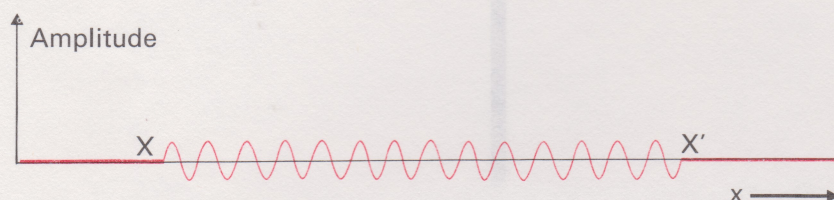


Figure 34 A wave train of finite extension.

This is because many different continuous wave trains must be superimposed in order to produce destructive interference beyond X and X' and so give the resultant wave train a finite extension. For a wave form to have only one characteristic wavelength, it is not sufficient that the distance between successive crests and troughs should be equal; the wave train must also extend to infinity. If one has to check that a passing wave train is infinitely long, one needs eternity to complete this measurement!

The energy-time uncertainty relation is extremely important in connection with atomic physics. In Units 6 and 7, you learnt that electrons can occupy different energy states within atoms, and that they stay in the higher energy states for only a limited time before spontaneously returning to a lower energy state with the emission of a photon. The energy of the photon represents the energy difference between the initial and final states of the electron and is equal to hf , where f is the frequency associated with the photon. But if an electron occupies a higher energy state for only a limited time, that means the total time available for making a measurement of the energy of the electron in that state is limited; any measurement of energy must apply to the state of the electron as it was at some instant within that restricted time interval. Equation 10 says that if one is able, by whatever means, to specify a restricted range of time during which the measurement must have been made (and the fact that what is being studied does not *exist* outside the time interval is undoubtedly one such means!), then the precision with which the energy can be defined is also limited.

Thus, energy measurements on electrons occupying the same high energy state in different atoms will not agree *precisely*. Certainly for most purposes

the electronic state can be thought of as having a simple characteristic energy—this assumption was implicit in the earlier Units—but strictly speaking this is not so.

Finally, in view of what has been said about imprecisely determined energies, we take another look at the law of conservation of energy. On the basis of this law, one expects that repeated measurements of the total energy of a system should yield a constant value. If a long time is available for each of these measurements, then each measurement will be very precise and, under these circumstances, it is indeed found that the law of conservation of energy holds. (Similarly, the law of conservation of momentum is also found to hold in quantum interactions where the momenta are determined precisely.) However, if the repeated energy measurements are made in rapid succession, the time available for each is limited and so the precision of each energy measurement is also limited. Under these circumstances, the repeated measurements do not give a steady constant value, but fluctuate with a spread characteristic of the uncertainty ΔE . Such readings do not contradict the law of conservation of energy—they indicate that it holds to within the limited precision permitted by the experiment. But note that this last set of observations does *not* prove that the law of conservation of energy must hold rigorously at all times. The shorter the time interval, the less one is able to say about what is happening to the energy of the system. Later, in Units 31 and 32, we shall introduce you to some highly successful theories of the nucleus which take advantage of the fact that it is impossible experimentally to specify the precise energy of the system over short periods of time.

29.5 Quantum Theory—Is it a Complete Theory?

Having now presented you with the basic ideas of quantum theory, we are in a position to describe a number of interesting consequences; indeed we have already indicated this much by giving you forward references to three of the remaining Units. But before applying the new ideas to a variety of situations, we thought you might like to learn of a famous controversy that split the world of physics in the 1920s and still has repercussions today. Its substance has little to do with your everyday life, or indeed with any scientific work in which you may be engaged. Nevertheless we hope you will find this excursion into philosophy an intriguing one.

Incidentally, in this section you will come across the names of several physicists, some of whom will be unfamiliar to you. Do not concern yourself unduly over the names—concentrate instead upon the arguments that are put forward.

An important concept involved in the controversy is that of causality. It is not easy to define this term in a way that would satisfy all philosophers, but we shall content ourselves with the following simple definition used by Planck:

There can be no more incontestable way to prove the causal relationship between any two events than to demonstrate that from the occurrence of one it is always possible to infer in advance the occurrence of the other. . . . An occurrence is causally determined if it can be predicted with certainty.

It is important to distinguish between two types of causality; these were first mentioned in section 29.3. You remember there was the type of causality that related to large statistical effects such as the passage of many photons through a slit, halving the volume of a gas, etc. Causality in this sense holds good and no one denies this. For example, an intense beam of monochromatic light passing through a slit causes a diffraction pattern of a certain size and intensity distribution, and this effect can be accurately predicted in advance. On the other hand, causality in the sense of predicting where an individual photon will go is known as *strict* causality. Quantum theory does not help us to make statements of a strictly causal nature.

two types of causality

29.5.1 The Copenhagen Interpretation

In a paper delivered at a conference in Brussels in 1927, Born and Heisenberg declared,

Quantum mechanics leads to accurate results concerning average values but gives no information as to the details of an individual event. The determinism which so far has been regarded as the basis of the exact sciences has to be given up. Every additional advance in our understanding of the formulas has shown that a consistent interpretation of the quantum mechanical formula is possible only on the assumption of a fundamental indeterminism. . . . We maintain that quantum mechanics is a complete theory; its basic physical and mathematical hypotheses are not further susceptible of modifications.*

* M. Born and W. Heisenberg, *Electrons et Photons—Rapports et Discussions due Cinquième Conseil de Physique Tenu au Bruxelles du 24 au 29 Octobre 1927*. Gauthier—Villars, Paris, pp. 160, 178 (1928).

This provocative statement in effect declared that quantum theory was to be regarded, in a sense, as the ultimate theory. It did not mean that quantum theory was incapable of improvement, refinement, and general adaptation to meet new situations; but it did express the view that quantum theory would never be replaced by a radically different theory founded on strict causality. According to this claim there was no hope of ever fulfilling Laplace's proud boast:

Give me the initial data on all particles and I will predict the future of the universe.

The uncertainty relations would always deny us the initial data from which to start. This view of quantum theory came to be known as the Copenhagen Interpretation (Copenhagen being the centre where people such as Heisenberg, Born and Bohr worked in the 1920s, while quantum theory was being developed).

Most, if not all, people at first find this view disquieting. It is not easy to abandon strict causality. It is somehow deep-rooted within us to think that the work of science will not be completed until it has established causal relationships which allow us to predict with certainty the outcome of all processes.

The Copenhagen school would argue that you cannot abandon something you have never had. There is no experimental evidence for strict causality and never has been. The only evidence for causality has been that relating to the weaker form of causality involving statistical processes, and the conclusions of quantum theory are in no way at variance with such experiments.* To maintain that all physical processes operate in a certain way (i.e. according to strict causality), when there is not the slightest evidence to support the view, is an act of faith strangely uncharacteristic of a scientist.

But, you may argue, if it is agreed that our *observations* of the world are subject to an imprecision imposed by the uncertainty relations, cannot one think of the world *itself* as being governed by causality. Here is Heisenberg's opinion:

In view of the intimate connection between the statistical character of the quantum theory and the imprecision of all perception, it may be suggested that behind the statistical universe of perception there lies hidden a 'real' world ruled by causality. Such speculations seem to us—and this we stress with emphasis—useless and meaningless. For physics has to confine itself to the formal description of the relations among perceptions.**

In support of this view he puts forward this argument:

We can say that physics is a part of science and as such aims at a description and understanding of nature. Any kind of understanding, scientific or not, depends on our language, on the communication of ideas. Every description of phenomena, of experiments and their results, rests upon language as the only means of communication. The words of this language

* The statistical nature of processes such as the build-up of a diffraction pattern by many photons is easily appreciated; but you may be wondering about a process such as the acceleration of an object (effect) produced by the Earth's gravitational force (cause)—what is 'statistical' about that? The answer to this will become clearer in Unit 32 when we introduce you to the idea of 'exchange forces'. Briefly, forces such as gravitational, electrical, and nuclear can be ascribed to the effects produced by the exchange of quanta between the interacting bodies. The acceleration of a body is therefore thought to be due to the accumulated recoils it experiences as the quanta are emitted and absorbed. Do not worry unduly about this at the moment; it is sufficient to know that to a quantum physicist even the acceleration of a body in a gravitational field incorporates a statistical element.

** W. Heisenberg, *Zeitschrift für Physik*, 43, p. 197 (1927).

represent the concepts of daily life, which in the scientific language of physics may be refined to the concepts of classical physics. These concepts are the only tools for an unambiguous communication about events, about the setting up of experiments and about their results. If, therefore, the atomic physicist is asked to give a description of what really happens in his experiments, the words 'description' and 'really' and 'happens' can only refer to the concepts of daily life or of classical physics. As soon as the physicist gave up this basis he would lose the means of unambiguous communication and could not continue in his science. Therefore any statement about what has 'actually happened' is a statement in terms of the classical concepts and—because of thermodynamics and of the uncertainty relations—by its very nature incomplete with respect to the details of the atomic events involved. Their demand to 'describe what happens' in the quantum-theoretical process between two successive observations is a contradiction *in adjecto*, since the word 'described' refers to the use of the classical concepts, while these concepts cannot be applied to the space between the observations; they can only be applied at the points of observation.*

This really takes us back to what was said in Unit 1. There it was pointed out that one of the great advantages man has over the other animals is the power of language. Like all animals he observes the world, but having done so he can communicate those observations to other men. Gradually over the ages a common fund of scientific knowledge is accumulated. But, in order to combine the results of the experiences of different persons, a common unambiguous language must be evolved. Such a language can only be ambiguous if it stems from some common experience or observation (perhaps not even then). A word used by a person X can only take on significance for person Y if it is related to some common aspect of X and Y's experience of the world, i.e. some common observation. Thus language has grown out of observation. Heisenberg argues in the passage quoted above that language is therefore only meaningful when applied to the acts of observation.

Thus we have one observation—an electron leaves an electron gun; and a second observation—the electron makes an image on a photographic plate on the other side of the barrier with the two slits in it. *But it is meaningless to argue which slit the electron went through.* The fact that it is difficult to visualize why an electron's collision with an atom in the rim of one of the slits should be affected by whether the second slit is open or closed is, according to the Copenhagen Interpretation, just one example of the trouble encountered when trying to apply concepts evolved *specifically* for describing observations to situations *in between* observations. Heisenberg declared:

It is a matter of personal belief whether such a calculation concerning the past history of the electron can be ascribed any physical reality or not.

In order to bring home to you just how much the processes of observation are inextricably linked with 'anything one tries to say about the micro-physical world, consider the following argument:

I take a sodium lamp and observe it with a counting device that produces an audible click the moment a photon arrives. Knowing how far the counter is from the source, and knowing that the photon travelled with the speed of light, I can then calculate how long it must have taken for the photon to travel from the source to the counter. By noting the time at which the click occurred, I can then calculate the instant at which the photon was emitted from the source. Note that I am now describing

* W. Heisenberg, *Physics and Philosophy*, p. 127, George, Allen and Unwin (1959).

something that took place when no observation was being made. Indeed, I can discard the counting device altogether and make the following general statement: at certain instants of time, the atoms in a sodium lamp emit photons even when they are not being observed.

Before reading on, think for a moment about that statement. Is it really an objective unambiguous statement about the real world in which the process of observation has been eliminated?

To see whether this statement is reasonable, consider the following behaviour of sodium lamps:

I take a sodium lamp and carry out an experiment with it involving interference, e.g. I might illuminate a Young's double slit and observe the fringes. This phenomenon I explain in terms of wave trains, one from each slit, travelling different paths HSP and HS'P (Fig. 35) and then

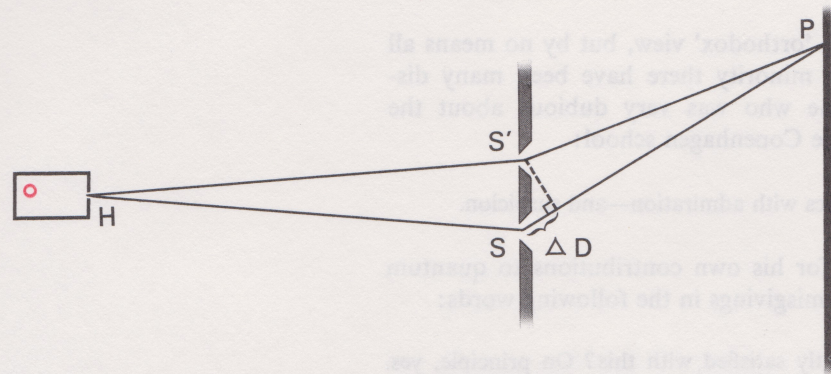


Figure 35 ΔD represents the difference in the path lengths HSP and HS'P.

interfering with each other at P. In order for the wave trains to recombine and produce interference at P, their lengths must be at least as long as the difference in the distances they travel, i.e. ΔD in Figure 35. (If they were not, the wave train travelling the shorter distance would have wholly arrived at P before the other had begun to arrive, and so they could not overlap and produce interference.) With such an arrangement, I make the source of light so weak that only the light from a single atom is allowed to pass through the apparatus at any given time. It is found that the phenomenon of interference has still to be taken into account in determining where the light is likely to go. Light emitted by a single atom is therefore in the form of a long wave train. In certain experimental arrangements, it has been found possible to produce interference between wave trains that have travelled paths differing in length by about 1 metre. The wave trains must therefore be at least this length. I finally conclude that for a sodium atom to produce a wave train of 1 metre length it must have emitted light over a period of time equal to 1 metre/velocity of light, i.e. $(1/3 \times 10^8)$ seconds, i.e. 3 nano seconds. Indeed, I can discard all reference to apparatus and make the following general statement: the atoms in a sodium lamp emit long wave trains of light over extended periods of time even when they are not being observed.

You have now been given two general statements about the behaviour of unobserved atoms in the sodium lamp. Do you see any conflict in accepting both of them?

In the first statement, the light was thought of as being emitted at a given instant in a sharp pulse (a photon), whereas in the second the light was emitted continuously over a long period of time (a wave train). You

cannot have it both ways. Which mental picture is right? It is impossible to say. *The atoms are not being observed*, so neither viewpoint can be verified.

In this way, you can see that the attempt to say something objective about the 'real' world divorced from the act of observation failed. *Both statements were still inextricably linked to a certain type of observation.*

Whether, therefore, one wishes to continue with mental pictures of what happens in between the observations in the face of such difficulties is a matter of personal preference. Some may find it helpful, others not. Whatever our preference, *science has nothing to say on the matter. Science is the study of our observations of the world, not of the world itself.* Moreover, the Copenhagen Interpretation holds that this is no temporary restriction to the activities of the scientist—a restriction likely to disappear with further advances in knowledge—but a fundamental limitation that will apply for all time.

Science is the study of our observations of the world, not of the world itself. This may be a fundamental limitation that will apply for all time

29.5.2 Dissenting views

Most physicists go along with this 'orthodox' view, but by no means all do. Indeed among the dissenting minority there have been many distinguished men. Einstein was one who was very dubious about the seemingly extravagant claims of the Copenhagen school:

I look upon quantum mechanics with admiration—and suspicion.

When receiving the Nobel prize for his own contributions to quantum theory, Schrödinger expressed his misgivings in the following words:

Will we have to be permanently satisfied with this? On principle, yes. On principle there is nothing new in the postulate that in the end exact science should aim at nothing more than the description of what can really be observed. The question is only whether from now on we shall have to refrain from tying description to a clear hypothesis about the real nature of the world. There are many who wish to pronounce such abdication even today. But I believe that this means making things a little too easy for oneself.

De Broglie began by trying to salvage strict causality, but was quickly won over to the Copenhagen Interpretation. In 1947 he wrote:*

I should also like to explain how, little by little, starting from the classical deterministic conception of physical phenomena which I adopted more from habit of mind than by philosophical conviction, I was led to range myself entirely on the side of the probability and indeterminist interpretation which Bohr and Heisenberg have given to the new mechanics. . . . Perhaps this 'confession' will have the further advantage of preventing certain investigators one meets with pretty frequently in these days, forcing themselves to turn back to the past, to question again the new conceptions of the physics of uncertainties . . . it is not out of mere wantonness that I have given up the traditional positions of classical physics but rather was I compelled and constrained to do so; perhaps this statement itself will show the incredulous how much these traditional positions have become impossible to defend.

And yet in spite of this statement, since 1951 he has again had doubts about the completeness of quantum theory:

At the level now reached by research in microphysics it is certain that the methods of measurement do not allow us to determine simultaneously

* L. de Broglie, *Physique et Microphysique*, Gauthier Villars (1947).

all the magnitudes which would be necessary to obtain a picture of the classical type of corpuscles (this can be deduced from Heisenberg's uncertainty principle), and that the perturbations introduced by the measurement, which are impossible to eliminate, prevent us in general from predicting precisely the result which it will produce and allow only statistical predictions. The construction of purely probabilistic formulae that all theoreticians use today was thus completely justified. However, the majority of them . . . have thought that they could go further and assert that the uncertain and incomplete character of the knowledge that experiment at its present stage gives us about what really happens in microphysics is the result of a real indeterminacy of the physical states and of their evolution. Such an extrapolation does not appear in any way to be justified. It is possible that looking into the future to a deeper level of physical reality we will be able to interpret the laws of probability and quantum physics as being the statistical results of the development of completely determined values of variables which are at present hidden from us. . . . One is glad to see that in the last few years there has been a development towards re-examining the basis of the present interpretation of microphysics. . . . It seems desirable that in the next few years efforts should continue to be made in this direction. One can, it seems to me, hope that these efforts will be fruitful and will help to rescue quantum physics from the cul-de-sac where it is at the moment.*

A leading advocate these days of the dissentient view is David Bohm. His opinion has been expressed as follows:**

. . . with regard to the current formulation of the quantum theory, we are led to criticize assumptions such as those of Heisenberg and Bohr, that the indeterminacy principle and the restriction to complementary pairs of concepts will persist no matter how far physics may progress into new domains. It should be clear, however, that in making such criticisms, it is not our intention to imply that the quantum theory is not valid or useful in its own domain. On the contrary, the quantum theory is evidently a brilliant attainment of the highest order of importance, a theory whose value it would be absurd to contest. . . . What we wish to stress here is, however, that the brilliant achievements of the quantum mechanics in no way depend on the notion that the features mentioned above (or any other features) of the current theory represent absolute and final limitations on the laws of nature. For all these achievements could equally well be obtained on the basis of the more modest assumption that such features apply within some limited domain and to some limited degree of approximation, the precise extent of which limits remains to be discovered. In this way we avoid the making of arbitrary *a priori* assumptions which evidently could not conceivably be subjected to experimental proof, and we leave the way open for the consideration of basically new kinds of laws that might apply in new domains, laws that cannot be considered if we assume the absolute and final validity of certain features of the theories that are appropriate to the quantum-mechanical domain.

Among the new kinds of laws that one is now permitted to consider if one ceases to assume the absolute and final validity of the indeterminacy principle, a very interesting and suggestive possibility is then that of a sub-quantum mechanical level containing hidden variables.

What is meant by 'hidden variables'? Let us explain with the help of an analogy. You no doubt recall seeing Brownian motion in Unit 5—the tiny smoke particles moving about in an agitated fashion. Suppose you

* L. de Broglie foreword to *Causality and Chance in Modern Physics*, by D. Bohm; Routledge, Kegan & Paul (1957).

** D. Bohm, *Causality and Chance in Modern Physics*, p. 100, Routledge, Kegan & Paul (1957).

knew nothing of the kinetic theory of gases—how would you explain the motion? You could not: the behaviour of the particles would seem strange and unpredictable. However, once the idea of rapidly moving molecules of air is introduced, the motion of the smoke particle is easily explained in terms of its collisions with these air molecules. A movement in any direction can be attributed to a statistical fluctuation in the number of impacts on opposite sides of the smoke particle; the phenomenon appears rational and predictable.

What is being suggested by Bohm is that there may exist hidden variables (analogous to the moving air molecules) at a sub-quantum level, behaving in accordance with strict causality. It is these variables that determine the *apparently* unpredictable behaviour of quanta (analogous to the motion of the smoke particles).

Matter exists at different levels of organization and of complexity, one of which is occupied by the organism called man. There is a great deal of evidence (some of which has been presented to you in this Course) that ways of describing the behaviour of matter (or our observations of matter) may be adequate and valid at one level but not at another.

For example, the discussion of species and populations in Unit 20 involves concepts appropriate to that particular level of organization. These are not the same concepts as those of Unit 15, which is concerned with cellular dynamics. This is in spite of the fact that populations are made of organisms and organisms are made of cells.

Likewise, the uncertainty principle is relevant to the diffraction of photons at a slit, but has little to do with the growth of the rabbit population in Australia. In this Unit great stress has been laid on the fact that physical variables (at least the ones known at present) can only say what happens at an observation and not what happens in between. But at the macroscopical levels of biology, geology and human practice quite the contrary is assumed. Descriptions given by these and other studies are based on the notion that objective processes actually take place 'in between' our observation of them (and, in the case of the Earth sciences often millions of years before there was any man to observe them!).

Thus we find qualitative changes in the language and concepts of science, corresponding to the qualitative changes in the behaviour of matter as we pass from one level to another. *It is near the boundaries between one level and another that the difficulties seem to arise.* In the Brownian motion example, the macroscopic object (smoke particle) is small enough to be affected by sub-macroscopic processes (motion of gas molecules), so it behaves in a 'strange' way.

Is there a level of organization of matter below that of the nuclear atom? Quantum mechanics describes adequately what happens at the microphysical level (the behaviour of electrons, atoms and molecules). But it is by no means clear that it is adequate when it comes to describing the structure of sub-nuclear matter. There may indeed be a more fundamental sub-microphysical level than that of nuclear and atomic physics, to which different and even more fundamental concepts than those of quantum mechanics may be applicable. The investigation of sub-nuclear matter is still at a very early stage, but even now there are some indications that we are once again near a new kind of threshold.

Classical mechanics has been found to be an adequate approximation at the macrophysical level, but at the microphysical level quantum mechanics has to be used. Is quantum mechanics, in its turn, only an approximation that is adequate at the microphysical level but inadequate at the sub-microphysical? And, if so, to what is it an approximation?

Even if the idea of hidden variables behaving in accordance with strict causality is not the correct one, a deeper understanding of Nature than

**it may be premature to abandon
the search for a deeper understanding
than that provided by quantum theory**

that at present provided by quantum theory may yet evolve along some other lines—*assuming steps are taken to seek one*.

As for the Bohr-Heisenberg view that one can never hope to use language evolved in the context of observation to describe what happens in between observation, this has also been challenged by Bohm. Here, his argument is that language can go further than mere vocabulary. Although the words of the basic language may originate in the context of common observation, language can use such words to evolve abstractions that go beyond observation. Such abstractions may one day yield a description of the real world independent of our observation of it.

Such, then, are the arguments used to counter the Copenhagen Interpretation.

29.5.3 Epilogue

What we have told you in this final section may well have come as something of a surprise. No doubt your recollection of school science left you with a feeling that in science everything is cut and dried. Science deals with facts and once a fact is established there is no argument about it. This of course is largely true. But, in this final section, the argument has not been about the factual achievements of quantum theory—these are indisputable—but about the areas into which scientific investigation and thought should now be directed. Decisions regarding the meaningfulness or otherwise of questions that might be asked of Nature can be very much a matter of personal opinion and judgement.

Summary of Unit 29

- (i) Electrons, nuclei, atoms and molecules exhibit observable diffraction effects.
- (ii) The diffraction patterns show that the wavelength associated with these forms of radiation is given by the de Broglie formula $\lambda = h/p$, where p is the momentum and h is Planck's constant.
- (iii) The lack of observable diffraction effects for large objects does not mean that de Broglie's formula cannot be applied to these objects. The wavelength expected on the basis of the formula would be too small to be noticed.
- (iv) The mathematics governing light propagation and that governing the motion of bodies have been shown to be equivalent. Thus, not only light (as was seen in Unit 28) but all forms of energy are propagated in accordance with wave theory.
- (v) If the position of a body is localized in space, the mathematical wave associated with it is also localized in space—it is a wave packet. Such a wave packet has a wavelength that cannot be precisely specified—it is some kind of average of the component waves that are used to build up the wave packet.
- (vi) The photo-electric effect shows that, in interactions between electromagnetic radiation and matter, energy is absorbed in discrete packets called photons. Each photon has energy hf , where f is the frequency of the radiation. You were reminded that discrete energy transfers were also characteristic of the emission as well as the absorption of electromagnetic radiation (Unit 6).
- (vii) Compton scattering shows that the momentum associated with a photon is hf/c , where c is the velocity of light in a vacuum.
- (viii) All interactions can be described in terms of discrete transfers of energy and momentum in accordance with the principles of conservation of energy and momentum.
- (ix) A study of the behaviour of a single quantum shows that the waves governing propagation behaviour are to be regarded as mathematical probability waves—the intensity of the wave at a given place determines the probability that the quantum will arrive at that place.
- (x) Because one can only predict probabilities for the various possible results of a measurement, it becomes impossible even in an idealized experiment to specify precisely the simultaneous values of the momentum and position of an object. Along each spatial direction there exists an uncertainty relation $\Delta p \cdot \Delta x \approx h$, where Δp and Δx are the uncertainties in momentum and position respectively. These relations were proposed by Heisenberg.
- (xi) A further uncertainty relation connects the uncertainty in the energy of a system, ΔE , with the time available for its measurement, Δt : $\Delta E \cdot \Delta t \approx h$
- (xii) Heisenberg's Uncertainty Principle draws attention to the fact that when an observer is confronted with a range of possible alternatives for measuring, say, the position and momentum of an object (the precision of these measurements being governed by the uncertainty relation), he can only gain the information appropriate to one of the alternatives. This is because each act of observation involves an unpredictable disturbance to the system.

- (xiii) Quantum theory deals only with *observations* of the world—not with a world divorced from the process of observation. According to the Copenhagen Interpretation this is no temporary restriction, but a fundamental limitation for all time—one cannot ever hope to go further and say something meaningful about what happens *in between* observations. In this sense it is claimed that quantum theory is complete. Not all physicists accept this claim.

Recommended Reading

R. P. Feynman, *Lectures on Physics*, Vol. 1, Chapters 26, 37 and 38. Addison-Wesley, 1963.

Quanta and Reality: a symposium, British Broadcasting Corporation, Third Programme lectures and discussion. Hutchinson, 1962.

Energy and Momentum Conservation in Compton Scattering

Figure 22 (p. 27) illustrates a collision between two bodies. One has mass M , and an initial velocity V , the other mass m , and is initially at rest. After the collision the body of mass M has been deflected through an angle θ and now has velocity v' . The second body recoils at angle ϕ to the incident direction and has velocity v .

Three equations can be set up as follows.

From conservation of energy:

$$\frac{1}{2} MV^2 = \frac{1}{2} Mv'^2 + \frac{1}{2} mv^2 \dots\dots\dots(11)$$

From conservation of momentum along the incident direction:

$$MV = Mv' \cos \theta + mv \cos \phi \dots\dots(12)$$

From conservation of momentum at right-angles to the incident direction:

$$0 = Mv' \sin \theta - mv \sin \phi \dots\dots(13)$$

Figure 21 (p. 27) illustrates Compton scattering, i.e. the scattering of a photon by an electron. The frequency of the incident radiation is f and that of the scattered radiation, f' . When momenta of hf/c and hf'/c are assigned to the two photons in accordance with Compton's proposal, and the energies of the photons are taken to be hf and hf' , it is found that the scattering can be described by three equations analogous to those given above:

$$hf = hf' + \frac{1}{2} mv^2 \dots\dots\dots(11')$$

$$hf/c = \frac{hf'}{c} \cos \theta + mv \cos \phi \dots\dots(12')$$

$$0 = \frac{hf'}{c} \sin \theta - mv \sin \phi \dots\dots(13')$$

Appendix 2 (Black)

Attempts to get round the Uncertainty Relation

First there is the approach that goes something like the following passage:

I shall use a slit to study the position of an electron. The slit is made as small as I like, so the electron's position is known. Then, instead of just looking at the general shape of the diffraction pattern and guessing a 'typical' value for the y component of momentum (i.e. $p \sin \theta$, where θ is shown in Figure 31, p. 39), I wait for one particular electron to strike the distant screen at a particular point L. By measuring the position of L, I can determine the angle LSO, i.e. the angle ϕ . Then $p \sin \phi$ is the *exact* value of the y component of momentum of this *particular* electron. There is no uncertainty in position and now no uncertainty in momentum.

Hazard a guess as to what might be wrong with this. (The fallacy lies in a basic misunderstanding as to how the uncertainty relation ought to be used.)

This argument involves two separate measurements. The first is a measurement of the position at the slit, S, the second a later measurement at the screen used for inferring what the momentum at the slit must have been. The fallacy is that the second measurement refers *back in time* to some former value of the momentum. The uncertainty relation stems from the concept of probability waves and these are concerned only with *prediction*. It is immaterial whether one cares to argue from what is seen on the screen to what happened earlier at the slit; the thesis being developed here is not concerned with that, but with the problem of predicting from measurements made at some instant what a future measurement will reveal. The uncertainty relation is concerned with prediction only and does *not* make statements about the past. In the experiment described above, the uncertainty relation says that the precise knowledge of position gained at the slit excludes the possibility of a precisely determined y component of momentum at the slit, and therefore excludes the possibility of *predicting* whereabouts on the screen the electron will go. Always be on your guard against misusing the uncertainty relation by applying it to measurements made in retrospect.

The next approach involves a genuine attempt to make a precise prediction:

Once again, I have a narrow slit so as to determine the position of the electron precisely. But this time I shall use something else to determine the electron's momentum. After it emerges from the hole, and before it strikes the screen, I shall take a look at it in a microscope (as in Fig. 36). Having located this second position, I know the direction in which it is travelling and so can predict where it will strike the screen.

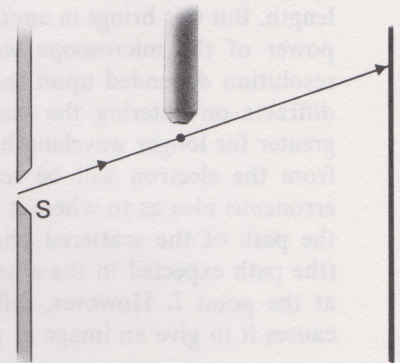


Figure 36 A microscope used in an attempt to locate the position of an electron after it emerges from the slit.

the uncertainty relation applies only to predictions

What is wrong with this approach?

Clue

If you use a microscope you will need to illuminate the electron to be able to see it. What will you use as illumination? Will the illumination have any effect on the electron?

The use of a microscope demands that the object be illuminated. This requires that light (i.e. at least one photon) should be scattered off it and into the microscope. But in a photon-electron collision there is an exchange of energy and momentum.

What name did we give to photon-electron collisions?

Refer back to section 29.2.2 if you do not know the answer.

This means that you are disturbing the electron; it will now go to some other place on the distant screen, not to the one you were trying to determine.

What kind of radiation would disturb the electron least?

You could of course reduce the momentum transferred to the electron by using photons of low momentum, i.e. light of exceedingly long wavelength. But this brings in another problem. In Units 2 and 28 the resolving power of the microscope was discussed; it was pointed out that the resolution depended upon the wavelength of the illumination. The light diffracts on entering the instrument and, of course, the diffraction is greater for longer wavelengths. Thus, a low momentum photon scattered from the electron will be so diffracted in a microscope as to give an erroneous idea as to where it has come from. In Figure 37, EL represents the path of the scattered photon. If it were to follow the optical path (the path expected in the absence of diffraction), it should give an image at the point I. However, diffraction at the entrance of the microscope causes it to give an image at point I'.

The question then becomes: given that we see an image I', where do we think the object was? With there being no way of estimating how much diffraction has occurred or whether the deviation was to left or right, the best that can be done is to assume no diffraction and project into space the line joining I' to the centre of the lens.* This gives an estimated position for the electron at E'. To this must be assigned an uncertainty depending on the size of the likely diffraction effects, and this uncertainty will be greater the longer the wavelength used. An infinitely long wavelength is needed in order not to disturb the motion of the electron at all, but the uncertainty in the position will then become infinitely great, and the microscope yields no information as to the position of the electron. So once again an attempt to circumnavigate the uncertainty relation founders.

The following is an example drawn from a wide class of approaches all founded on the same basic type of flaw.

I make the slit in Figure 31 (p. 39) as small as I like so as to fix the position. The barrier with the slit in it is free to move in the y direction. If it is initially at rest, it will be set moving with velocity, v , in the y direction as a result of the momentum imparted to it by the electron as the electron acquires sideways momentum at the slit. If the mass of the barrier is m ,

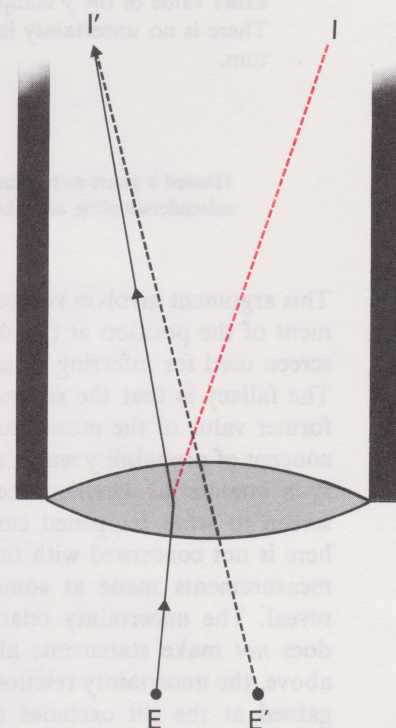


Figure 37 A photon reflected from the electron at position E does not go to the expected image position I because it suffers diffraction at the opening of the microscope. It goes instead to form an image at I'. The object position corresponding to I' is E', not E, so an uncertainty in the position of the electron is introduced.

* If you have forgotten why we use this procedure for relating object and image positions read the discussion of Figure 58 in Appendix 5 of Unit 2.

then the momentum is mv . Because momentum is conserved in the y direction, as in any other direction, the momentum of the barrier mv , must be equal and opposite to the y component of momentum of the electron, $p \sin \phi$. Therefore $mv = p \sin \phi$, hence ϕ is determined. I now know precisely the angle ϕ and the position of the electron as it emerged from the slit. Admittedly I determined the momentum of the electron by a second measurement in retrospect but in making this estimate I did not disturb the momentum of the electron itself so I can now go on to *predict* whereabouts on the screen the electron will go.

This is indeed a clever argument. The electron is not disturbed after it passes through the slit, and it is a genuine attempt at prediction. If you find it difficult to see what is wrong with it you may take comfort from the fact that many a seasoned physicist fails to see the weakness too!

Clue

The barrier with the slit in it is said to be at rest initially. This means its initial momentum is precisely known to be zero. What does that say about our knowledge of the position of the barrier?

The estimate of the sideways momentum imparted to the barrier is based on the difference between the initial and final momenta of the barrier in the y direction. These must both be known precisely if the final result is to be precise. But how does one *know* the precise initial value of the momentum of the barrier or indeed the position of the slit? It is implicitly *assumed* in the wording of the argument that this knowledge has been gained through performing a preliminary experiment. Such a preliminary experiment might consist of shining a light on the barrier on two separate occasions. If, for example, the screen is seen not to move between the two observations, then it can be concluded that the initial momentum of the barrier is zero. But how can one be sure that the second time the barrier was illuminated it did not acquire some momentum from the photons? If it did, then the initial momentum of the barrier when the electron arrives at the slit would not be zero. The only way to be sure that the state of rest of the barrier is not disturbed by the second observation is to use light of long wavelength. This in its turn means that the *preliminary experiment is incapable of determining precisely the initial position of the barrier—and hence the position of the slit in the y direction*. The reason is the same as that encountered in the previous example, where it was found that long wavelengths imply large diffraction effects in the viewing instrument (e.g. a microscope) and hence uncertainty in the position of the object viewed. A precise knowledge of the initial momentum of the barrier, combined with only an imprecise knowledge of the position of the slit, brings us *no* closer to making a precise prediction.

From this example, you learn that before considering the experiment proper (in this case the electron arriving at the slit), you have to be quite sure you can set up the apparatus as described. In this example you could not have done so. A preliminary measurement designed to ascertain the precise momentum of the barrier would inevitably have left the slit with an uncertain position. An alternative kind of preliminary experiment designed to ascertain the precise position of the slit (using short wavelength illumination) would have lost all knowledge of the momentum of the barrier. You have to be very careful to spell out exactly what the function of each part of your apparatus is. If, as in our earlier examples, the slit is used to specify the position of the electron, then it has to be firmly understood that the slit's position is known only as a result of it

always be clear in your mind what preliminary measurements might be necessary, and examine carefully whether they can provide the information you require in order to start the experiment proper

being *fixed*, and consequently incapable of recoiling and giving information about the electron's momentum. On the other hand a momentum-measuring barrier designed to recoil, cannot give a precise fix on the position of the electron. *You cannot have it both ways*—you must make your choice and stick to it.

We shall not pursue these arguments any further. You are, of course, free to continue the search for ways round the uncertainty relation if you so wish—but you are advised not to spend too much time on it!

This is indeed a clever argument. The electron is not disturbed after it passes through the slit, and it is a genuine attempt at precision. If you find it difficult to see what is wrong with it you may wish to consult the fact that many a second-order optical filter is used in the experiment.

One
The barrier with the slit is not a very ideal one. The reason is that the momentum is precisely known to be zero. What does that tell you about the knowledge of the position of the barrier?

The estimate of the sideways momentum imparted to the barrier is based on the difference between the initial and final momenta of the barrier in the y direction. These must both be known precisely if the final result is to be precise. But how does one know the precise initial value of the momentum of the barrier or indeed the position of the barrier? It is implicitly assumed in the working of the argument that this knowledge has been gained through performing a preliminary experiment, such as a preliminary experiment might consist of shining a light on the barrier on two separate occasions. If, for example, the screen is seen not to move between the two observations, then it can be concluded that the initial momentum of the barrier is zero. But how can one be sure that the second time the barrier was illuminated it did not acquire some momentum from the photons? It is difficult then to find the initial momentum of the barrier when the electron arrives at the slit would not be zero. The only way to be sure that the state of the barrier is not disturbed by the second observation is to use light of long wavelength. This in its turn means that the preliminary experiment is incapable of determining precisely the initial position of the barrier—and hence the position of the slit in the y direction. The reason is the same as that encountered in the previous example, where it was found that long wavelengths imply large diffraction effects in the viewing instrument (e.g. a microscope) and hence uncertainty in the position of the object viewed. A precise knowledge of the initial momentum of the barrier, combined with only an imprecise knowledge of the position of the slit, brings us no closer to making a precise prediction.

From this example you learn that before considering the experiment except in this case the electron arriving at the slit, you have to be quite sure you can set up the apparatus as described. In this example you could not have done so. A preliminary measurement designed to ascertain the precise momentum of the barrier would inevitably have left the slit with an uncertain position. An alternative kind of preliminary experiment designed to ascertain the precise position of the slit (using short wavelength illumination) would have lost all knowledge of the momentum of the barrier. You have to be very careful to spell out exactly what the function of each part of your apparatus is, as in our earlier examples. The slit is used to specify the position of the electron, then it has to be firmly understood that the slit's position is known only as a result of it

always be clear in your mind what preliminary measurements might be necessary, and estimate carefully what they can provide. The information you require in order to start the experimental proper

Appendix 3 (Black)

The Derivation of the Energy-Time Uncertainty Relation

The momentum, p , of an object can be expressed in terms of its kinetic energy, T , in the following way:

$$\begin{aligned} mv &= p \\ (mv)^2 &= p^2 \\ 2m(\frac{1}{2}mv^2) &= p^2 \\ T &= p^2/2m \dots\dots\dots(14) \end{aligned}$$

The uncertainty in kinetic energy, ΔT , has now to be expressed in terms of the uncertainty in momentum, Δp . Suppose p is increased by the value of its uncertainty to $(p+\Delta p)$, T will be correspondingly increased to $(T+\Delta T)$, and there will be a relation analogous to equation 14 connecting these new values of momentum and energy.

$$\begin{aligned} T + \Delta T &= \frac{(p+\Delta p)^2}{2m} \\ T + \Delta T &= \frac{p^2}{2m} + \frac{2p\Delta p}{2m} + \frac{(\Delta p)^2}{2m} \dots\dots\dots(15) \end{aligned}$$

Replacing T on the left-hand side of equation 15 by the expression contained in equation 14.

$$\begin{aligned} \frac{p^2}{2m} + \Delta T &= \frac{p^2}{2m} + \frac{p\Delta p}{m} + \frac{(\Delta p)^2}{2m} \\ \Delta T &= \frac{p\Delta p}{m} + \frac{(\Delta p)^2}{2m} \dots\dots\dots(16) \end{aligned}$$

If Δp is small, the second term on the right-hand side is small compared with the first and can be ignored. Therefore:

$$\Delta T = \frac{p\Delta p}{m} \dots\dots\dots(17)$$

If the shutter in Figure 33(b) (p. 43) remains open for a time Δt , then the measurement can be considered to take place at some moment within that interval. Thus Δt is the uncertainty in the time of the measurement. If the object moves with velocity v , then the distance it travels in time Δt is given by

$$\Delta x = v \cdot \Delta t \dots\dots\dots(18)$$

So if it is known that the object arrives at the slit *sometime* within the interval Δt , then one can say that at any *given* instant within the range Δt , the uncertainty in the position of the object is Δx .

We now write down an expression for the product $\Delta T \cdot \Delta t$:

$$\begin{aligned} \Delta T \cdot \Delta t &= \frac{p\Delta p}{m} \cdot \frac{\Delta x}{v} \\ &= \frac{p}{mv} \Delta p \cdot \Delta x \end{aligned}$$

Thus, because $p=mv$

$$\Delta T \cdot \Delta t = \Delta p \cdot \Delta x \dots\dots\dots(19)$$

But $\Delta p \cdot \Delta x \approx h$

Therefore

$$\Delta T \cdot \Delta t \approx h \dots\dots\dots(20)$$

The total energy of an object is the sum of its kinetic energy, T , and its potential energy, U :

$$E = T + U \dots\dots\dots(21)$$

If the object is 'free' so that its potential energy is zero (or at least independent of x), the uncertainty in the kinetic energy, ΔT , gives rise to an equal uncertainty in the total energy ΔE . Therefore, substituting ΔE for ΔT in equation 20, one obtains the energy-time uncertainty relation referred to in the text as equation 10:

$$\Delta E \cdot \Delta t \approx h \dots\dots\dots(10)$$

Self-Assessment Questions

Section 1.1

Question 1 (Objective 1)

Planck's constant is:

- (i) a number without dimensions
- (ii) a quantity of action
- (iii) a quantity of momentum
- (iv) a quantity of energy.

Question 2 (Objective 2)

If in Figure 2 (p. 10) the separation of the atoms in the surface of the crystal is 2.15×10^{-10} m, and the direction of the second diffraction maximum makes an angle of 55° to that of the initial electron beam incident normally to the surface, calculate the wavelength of the electrons. (You may ignore effects arising from reflection at the lower layers of atoms.)

Question 3 (Objective 3)

A proton has a diameter 3×10^{-15} m and a mass of 1.673×10^{-27} kg. Planck's constant is 6.62×10^{-34} J s, and the velocity of light 3×10^8 m s $^{-1}$. The de Broglie wavelength of a proton is:

- (i) 1.5×10^{-15} m;
- (ii) 3×10^{-15} m;
- (iii) 3.95×10^{-7} m;
- (iv) 1.316×10^{-15} m;
- (v) known, but none of the above;
- (vi) unknown owing to insufficient data.

Section 1.4

Question 4 (Objective 4)

When Maupertuis' Principle of Least Action was being applied to the possible paths a particle might take between two points, it was noted that the energy of the particle was considered to be constant over each path. When Fermat's Principle of Least Time was being applied to the possible paths light might take through media of varying refractive index, the following was kept constant over these possible paths:

- (i) the wavelength
- (ii) the frequency
- (iii) the speed

(a)	(b)
True	False

Question 5 (Objective 4)

According to Fermat's Principle of Least Time, light takes the path of shortest time between two points. Figure 38 shows light from an object O travelling by *many* paths to an image I: How do you account for there being more than one path?

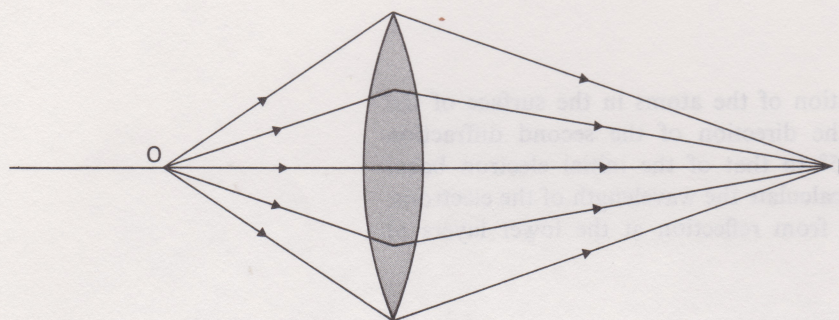


Figure 38 Diagram for SAQ 5.

Section 1.5

Question 6 (Objective 7)

Two continuous wave trains of almost equal wavelengths, λ and $\lambda + \Delta\lambda$, are superimposed to produce wave packets. What if anything would happen to the length of the packets if $\Delta\lambda$ were increased?

The length of the packets would:

- (i) increase;
- (ii) decrease;
- (iii) remain unaltered.

Section 2.2

Question 7 (Objective 5)

- (i) The energy of a photon is proportional to frequency.
- (ii) The momentum of a photon is proportional to frequency.
- (iii) The energy of a photon in a piece of plastic is inversely proportional to the wavelength of the radiation in a vacuum.

(a)	(b)
True	False

Question 8 (Objective 5)

This question is concerned with the photo-electric effect. Tick the appropriate statements.

The maximum energy of the emitted electrons depends upon:

- (i) the frequency of the incident light;
- (ii) the intensity of the incident light;
- (iii) the nature of the metal surface.

(a)	(b)
True	False

The rate of emission of photo-electrons depends upon:

- (iv) the frequency of the incident light;
- (v) the intensity of the incident light;
- (vi) the nature of the metal surface.

(a)	(b)
True	False

Section 4.0

Question 9 (Objective 6)

The fact that there are regions of zero intensity in the distribution on the right-hand screen of Figure 31 (p. 39) is an indication of the following:

(a)	(b)
True	False

- (i) Certain values of the y component of momentum are never imparted to the radiation at the slit, S.
- (ii) The area of the slit, S, is not uniformly illuminated by the radiation incident on it.

Question 10 (Objective 8)

In making a measurement, the system being observed is disturbed—this is a new idea introduced by the uncertainty principle of quantum theory. Do you agree that this is a new idea?

Sections 1.4, 3.0, 5.0, 5.1, 5.2

Question 11 (Objective 9)

This is the problem set by Professor Pentz in the TV programme. It concerns Young's double-slit arrangement.

You will remember how, in the case of quanta, it was impossible to tell which of the two slits the quantum went through (section 3.0). Indeed, according to the Copenhagen Interpretation, the question 'which slit does the quantum go through?' is meaningless, because it refers to something that happens *in between* observations (section 5.1). You will also remember that we have drawn a close parallel between the propagation behaviour of quanta and that of macroscopic bodies (section 1.4). The questions raised in the final sequence of the TV programme can be stated as follows:

If the behaviour of quanta and macroscopic bodies are so similar, and if it is impossible to tell which of the two slits a quantum goes through, how is it you are confident you know which of two doorways the Stannard-type radiation passes through? Are you, in fact, confident? Or is the question 'Which doorway did Dr. Stannard go through?' also, strictly speaking, meaningless?

Self-Assessment Answers and Comments

Question 1

Answer (ii)

Comment

Action has been presented to you in the form momentum $\times$ distance.
This has dimensions

$$\left(\frac{ML}{T}\right) \times L = ML^2T^{-1}$$

Planck's constant has units: joule second, i.e. the same dimensions as energy $\times$ time,

i.e.
$$\left(\frac{ML^2}{T^2}\right) \times T = ML^2T^{-1}$$

h is in fact referred to in some books as 'Planck's quantum of action'.
Refer to *HED*, section 4, if you are not clear about arguments based on dimensions.

Question 2

Answer 0.88×10^{-10} m

Comment

From equation 1,
$$\lambda = \frac{d \sin \theta}{n}$$

$d = 2.15 \times 10^{-10}$ m; $\theta = 55^\circ$; and $n = 2$ because it is the second diffraction maximum

$$\therefore \lambda = \frac{2.15 \times 10^{-10} \sin 55^\circ}{2} = 0.88 \times 10^{-10} \text{ m}$$

Question 3

Answer (vi)

Comment

You have not been given the velocity of the proton and hence cannot substitute for its momentum, p , in the equation $\lambda = h/p$.

Question 4

Answer (i) b, (ii) a, (iii) b.

Comment

The frequency is kept constant. This means that the energy of the photon (hf) is considered fixed—just as was the case for the particle.

Question 5

Answer As light can travel by many paths, the time taken to traverse any of them must be the same. The slowing-down of the light as it passes through the thick glass at the centre of the lens compensates for the longer distances that have to be travelled by the rays passing through the edge of the lens.

Question 6

Answer (ii)

Comment

If $\Delta\lambda$ were increased, then the difference between the two wavelengths would be greater and the two wave trains would get out of step more quickly—thus giving shorter wave packets.

Question 7

Answer (i) a, (ii) a, (iii) a.

Comment

Note in connection with (iii) that the energy of a photon, hf , depends upon the frequency, and this does not change when the radiation passes from vacuum into a medium.

Question 8

Answer (i) a, (ii) b, (iii) a, (iv) a, (v) a, and (vi) a.

Comment

You may have overlooked (iv) a and (vi) a. If the change in the metal surface makes it more difficult for the electrons to get out, then some of the electrons formerly emitted with low energy may now not be able to emerge—thus the rate of emission of photo-electrons is reduced. If the frequency were to be reduced below the cut-off frequency, f_0 , then no electrons would be emitted. Thus a change in frequency has an effect on the rate of emission.

Question 9

Answer (i) a and (ii) b.

Comment

The position of a photon on the screen is a measure of its sideways momentum at the slit. If there is zero probability of the photon going to certain regions of the screen, then there must be zero probability of receiving the appropriate value of the sideways momentum to take it to one of these regions.

Question 10

Answer It is not a new idea. It has long been recognized that the process of observation can interfere with the system being observed. What is new in quantum theory is that such disturbances involve an element of *unpredictability*—it is impossible to correct for them.

Question 11

Discussion This is no pat answer to the question of whether or not it is possible to tell which door Dr. Stannard went through on the TV programme. So, if you arrived at a ready answer with no trouble, we suggest you reconsider the question before reading further—you may not have thought about it as deeply as you ought!

The first point to note is that the de Broglie wavelength of a person moving at walking pace is minute compared to the dimensions of doors or their separation. (Recall the example of the billiard ball of section 29.1.3. Also note we are talking of the de Broglie wavelength and not the physical dimensions of the person.) When the wavelength of radiation (be it electron radiation or Stannard-type radiation) is much smaller than the separation of the double slits, it becomes possible to eliminate any interference between radiation from the two slits by only illuminating one of the slits as in Figure 39. This would be the basis upon which some people would argue that it *was* possible to tell which slit the radiation passed through.

one viewpoint

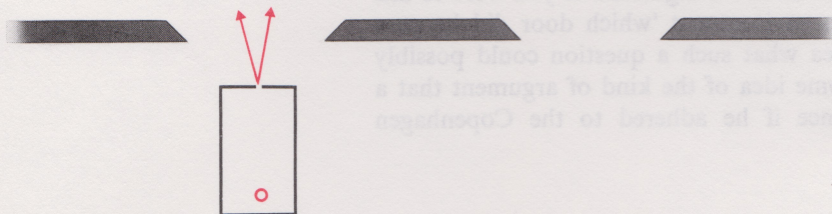


Figure 39 A drawing associated with the answer to SAQ 11.

But if it is held that in this extreme case of short wavelengths the answer is known, whereas in the other extreme case of long wavelengths (long that is compared to the separation of the slits) the answer is not known, what is to be done about an intermediate case where there may or may not be a vestige of interference with radiation from the second slit? How does one make a neat division between the two cases?

It could be argued that there is no need for a neat division. The probability that the radiation passes through a particular slit depends upon the intensity of the radiation illuminating that slit. In one extreme case, the probability favours one slit almost to the total exclusion of the other; in the other extreme case, where both slits are equally illuminated, there is a 50 per cent chance for each slit. In between these extremes there are cases of intermediate probability.

According to an entirely different viewpoint, it could be claimed that in no case (including the one shown on TV) was it known which slit the radiation passed through—that the question was meaningless. According to this view it would first be necessary to define what is meant by the act of ‘passing through the doorway’. Presumably there would be an imaginary line separating one side of the doorway from the other. ‘Passing through the doorway’ would then be the crossing of this line by some chosen central point of the person’s body. But the act of observing the person is not continuous. Observation consists of the receipt of a stream of isolated photons. Some of these photons would indicate that the central point of the body at a given instant was on one side of the line and the others would show it to be on the opposite side. (We gave you a clue to this in the TV programme by making the screen go blank at the critical moment when Professor Pentz walked through the doors. Of course, watching someone on television brings in an additional source of discontinuity—you are seeing separate pictures at the rate of 50 per minute (Unit 2). Quite apart from this added complication though, our normal process of observation is discontinuous.) No photon could show you the body’s central point smoothly crossing the line from one side to the other. *Continuous processes are mental abstractions, not observations.* Such abstractions are, of course, used freely in everyday life and no one would think of dispensing with them in that context. It is only when they are applied to the microphysical domain that inconsistencies arise and the dubious nature of such abstractions becomes manifest.

It is interesting to reflect that if Planck’s constant were to change by some vast factor, so that photons for example would strike blows like cannon balls, a description of everyday life would be utterly different to what it is now. It would then be quite obvious that one’s observations of Dr. Stannard consisted of nothing more than isolated ‘snapshots’ of him on one side or other of the double-door arrangement. If you were to ask a person brought up in such an environment ‘which door did he pass through?’ he would have no idea what such a question could possibly mean! This at least gives you some idea of the kind of argument that a physicist would probably advance if he adhered to the Copenhagen viewpoint.

a second viewpoint

Acknowledgements

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ILLUSTRATIONS

PROFESSOR C. JÖNSSON, Fig. 1(a); LIBRAIRIE LAROUSSE, Fig. 11; PROFESSOR A. B. PIPPARD, Cavendish Laboratory, Fig. 3.

Notes

